Thick conducting shells at $R$, $2R$, $3R$

More from inner shell to middle, what happens?

\[ \Phi = \frac{kQ}{r} + \text{const.} \]

\[ \Phi = \frac{kQ}{r} \]

\[ \Phi = \frac{kQ}{r} - \frac{kQ}{2R} \]

Note $E$ does not go to 0 at $2R$ smoothly like when $r \to \infty$. It has a "jump" at the charge since $\nabla \cdot E = \rho / \epsilon_0$

$\Rightarrow \frac{\partial E}{\partial r} = \rho / \epsilon_0$

In spherical coords $\nabla \cdot E = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 E_r)$

$\frac{1}{r^2} \frac{\partial}{\partial r} (kQ) = 0$

Except where you have a charge, there you get a spike (discontinuity in the derivative).
If conductor not spherical, distribution not uniform.

\[ q = 8 \pi \epsilon_0 R^2 \]

For sphere, \( q = \text{const} \). Charge distribution on surface.

\[ E = \frac{q}{4 \pi \epsilon_0} \]

\( E \) is negligible on surface of conductor.

No E-field in conductor.

Draw dx, dS, s/dS.
All conductors:

Put -q on middle shell,

What happens?

Now, connect inner + outer sphere together,

Net -q on middle

\[ q = \frac{(a-c) + q}{2} \]

We need -charge outside (2R) is +charge on outside of (R)

Surface, so potential change from R to 3R surfaces

as \( \mathbf{E} \) points out from \( R+2R \), it points in from \( 2R \) to \( 2R \)

\[ \int \frac{kq}{r^2} \, dr = \frac{2\pi}{3} \times \frac{1}{2} (a-c) \]

\[ (-kq) \frac{1}{r} \bigg|_{R}^{2R} = -k(-c-a) \frac{1}{r} \bigg|_{R}^{2R} \]
\[-k_B \left( \frac{1}{2r} - \frac{1}{12} \right) = -k_B \left( \frac{Q - q}{2r} \right) \left( \frac{1}{2r} - \frac{1}{3r} \right)\]

\[\frac{k_B}{2r} = k_B \left( \frac{Q - q}{2r} \right) \frac{1}{6r}\]

\[q = q - 8\]

\[q = 0\]

\[q = 8q / 4\]

\[E = \frac{kQ}{4 \pi r^2}\]

\[E = -\frac{3}{4} \frac{kQ}{r^2}\]

Check:

\[\sum_{r} \frac{kQ}{4 \pi r^2} dr = \frac{kQ}{4} \left( \frac{1}{r} \right)_{2r} = -kQ \left( \frac{1}{2r} - \frac{1}{r} \right) = \frac{kQ}{8r}\]

\[\sum_{r} -3 \frac{kQ}{4 \pi r^2} dr = \frac{3}{4} kQ \left( \frac{1}{r} \right)_{2r} = \frac{3}{4} kQ \left( \frac{1}{2r} - \frac{1}{3r} \right) = \frac{3}{4} kQ \left( \frac{1}{2r} \right)\]

\[z = -kQ / 8r\]

Equal in mag, opposite in sign, all good!
Image charges

- Charged Conductor

Field at point charge
near point charge
Field at surface at conductor
how to solve?
Certain simple geometries - image charge

Place an image charge so that fields are at conductor surface.

\[ \overset{\text{cond.}}{\text{plane}} \]

\[ \overset{\text{cond. sphera}}{\text{(?)}} \]

\[ E_2 = \frac{-2kQ}{r^2 + h^2} \cos \theta \]

\[ z = \frac{-2kQh}{\sqrt{r^2 + h^2}} \]

\[ \sigma = \varepsilon_0 E_2 = -\frac{2\varepsilon_0 Qh}{4\pi \varepsilon_0 (r^2 + h^2)^{3/2}} \]

\[ \int_{\text{plane}} \sigma \, dA = \int_{\text{plane}} \frac{Qh}{2\pi (r^2 + h^2)^{3/2}} \]

\[ q_{\text{plane}} = \int_{0}^{\infty} \frac{-Qh (z)}{r^2 + h^2} \left( \frac{1}{(r^2 + h^2)^{3/2}} \right) dz = \frac{Qh}{(4\pi \varepsilon_0)^{1/2}} \int_{0}^{\infty} \frac{-Qh}{(r^2 + h^2)^{3/2}} \]

Should expect that!
Capacitors - store charge

\[ Q = CV \]

\[ C = \frac{Q}{V} \]

Use \( d \ll \frac{1}{A} \) approximation.

Constant field inside

\[ \sigma = \frac{Q}{A} = \varepsilon_0 E \]

\[ V = \Delta Q = \int E \cdot d = \frac{\sigma d}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A} \]

\[ C = \frac{Q}{V} = \frac{Q}{\left( \frac{Q}{\varepsilon_0 A} \right)} = \frac{\varepsilon_0 A}{\varepsilon_0} \]

For same \( V \) (same \( E \) field)

get more \( C \), for larger \( A \), smaller \( d \)

\( Q = CV \) is general, can apply to coaxial cylindrical shells or concentric spheres

\[ E = \frac{Q}{4\pi \varepsilon_0 R^2} \]

\[ \Delta Q = \frac{\varepsilon_0}{4\pi \varepsilon_0 R} \left( \frac{1}{R} - \frac{1}{R'} \right) = \frac{Q}{8\pi \varepsilon_0 R} \]

\[ C = \frac{Q}{V} = \frac{Q}{\Delta Q \varepsilon_0 R} = \frac{1}{8\pi \varepsilon_0 R} \]

Notice

\( Q \) is constant + \( E \) & geometry factors

What about cylindrical shells close together??
1. Consider the system of 3 concentric spheres with the outer and inner connected. What is the capacitance of the system? (See notes, pages 3-4) Calculate $C = Q/V$. What is the right $Q$? What is the right $V$?

2. Consider a system of a long narrow cylinder of conducting radius $R$ and length $L$, inside a cylindrical shell of radius $R_s$ and length $L$. Put a charge $+Q$ on the cylinder and $-Q$ on the shell. What is the potential difference and the capacitance of the system?

3. There are 2 infinite conducting planes, one defined by the $yz$ axes and one by the $xz$ axes. A charge $Q$ is placed on the $xy$ plane, e.g., at $x = 1$. What and where are the image charges?

4. Two spheres of radius $R$ are centered a distance $d > 2R$ apart. Calculate the capacitance of the system. Assume the charges on the two spheres are $+Q$ on one and $-Q$ on the other. What then is the potential difference $V = \Delta \phi$, and the capacitance $C = Q/V$?
Show that the surface \( r = R \) has \( \phi = 0 \).
(Thus, if you put a spherical shell conductor at \( R \),
 each of the charges would be the image of the other —
 the field configuration would be the same.)

It is sufficient to do this for an arbitrary point on the
surface \( x = R \cos \theta, y = R \sin \theta, z = 0 \).