Units: MKS

- Atoms
- \( p, n \)  
  \( \rightarrow \)  
  \( e^- \)
- Strong, Weak, Force
- Gravity
- \( e^- \)

World around us is \( E \cdot M \) (GM) in a flat gravitational field.

Ideas worked out in 1800s.

- No relativity, no curved space, no GM.

Start with electromagnetism - charges fixed in place, don't move.

Specify charge configuration. What are forces?

\( \vec{F}_{1 \rightarrow 2} = \frac{k Q_1 Q_2}{r_{21}^2} \)

\( k = \frac{1}{4\pi \varepsilon_0} \approx 9 \times 10^9 \text{N m}^2/\text{C}^2 \)

Parallel to line (symmetry?)

\( 1/r^2 \) - why?

Like charges repel, unlike charges attract.

Superposition - multiple forces simply add.
\[ F_{2 \text{TOTAL}} = F_{1 \text{ON } 2} + F_{2 \text{ON } 2} \]

What changes get \( \vec{F} \) in different quadrants, for \( Q_2 > 0 \)?

All electrostatics solved in principle. Add all forces

\[ F_{\text{ON}_i} = \sum_j \frac{k Q_i Q_j}{r_{ij}^2} \]

Typical problems: given \( \vec{F}_i, Q_j \) find \( F_{\text{ON}_i} \)

given all but 1 of \( \vec{F}_i, Q_j \) and \( F_{\text{ON}_i} \), find missing one

In classical physics, individual charges \( \rightarrow \) charge distribution.

\[ \sum_i \rightarrow S \]

\[ F_{\text{ON}_i} = \int \int \frac{k Q_i \rho (\vec{r})}{|\vec{r}_i - \vec{r}|^2} \]

not analytically solvable in general

best done by computer

But in class we usually pick geometries you can calculate analytically
Electric field

\[ \vec{F}_{Q_2} = \frac{k Q_1 Q_2 \hat{r}_2}{r_2^2} \]

Put Q_1 at origin for simplicity

Convenient to write \( \vec{F}_{Q_2} = Q_2 \vec{E} \)

\( \vec{E} = \frac{\vec{F}_{Q_2}}{Q_2} \)

why? Bookkeeping?

No fields are real

As: \( \vec{E} \) (and \( \vec{B} \)) fields carry energy

Superposition \( \vec{E}_{\text{total}} = \vec{E}_1 + \vec{E}_2 + \ldots \)

Field from a ring

Charge \( Q \) spread on a circular ring

\[ p = \frac{Q}{2\pi R} \]

What is \( \vec{E} \) field in the center of the ring?

Brute force, integral, or symmetry argument?

If the world looks the same in multiple directions, the field cannot be in any of them.
\[ E = \frac{kQ}{r^2} \quad \text{for point charge} \]

\[ E_z = \frac{1}{4\pi\varepsilon_0} \frac{Q}{2\pi R} \frac{1}{(R^2 + z^2)^{3/2}} \]

\[ E_z = \frac{Q}{4\pi\varepsilon_0} \frac{z}{(R^2 + z^2)^{3/2}} \]

\[ z \to 0 \quad E_z \to 0 \]

\[ z \to \text{large} \quad E_z \to \frac{Q}{4\pi\varepsilon_0} \frac{1}{R^2} \]

Textbook covers hemisphere, at \( z = 0 \). Similar but more algebra.

**Flux**: \( \int \vec{E} \cdot d\vec{A} \), amount of field through a surface

Water analogy

Not just \( \vec{E} \cdot A \), but \( \vec{E} \cdot d\vec{A} \)
What about flux through closed surface? into/out of a volume

If no sources (charges), no build up, no net flux, \( \Phi = 0 \)

Incompressible water analogy

\[ \int \vec{E} \cdot d\vec{A} = -EA \text{ left} \]
\[ 0 \text{ sides} \]
\[ +EA \text{ right} \]
\[ 0 \text{ total} \]

Gauss's Law \( \Phi = \frac{\int_{\text{closed surface}} \vec{E} \cdot d\vec{A}}{\varepsilon_0} \)

Point charge "proof": spherical shell centered on charge

\[ \Phi = \frac{Q}{\varepsilon_0} \]

Is the answer different if we change the position of the charge or shape of the surface?

Once you accept for 1 point charge, superposition \( \Rightarrow \) valid for any charge distribution.
popular charge distributions for Gauss's Law

sphere of charge: constant density
\[ \rho = \frac{Q}{4\pi R^2} \]

Outside: same field as point charge Q:

\[ \oint \mathbf{E} \cdot d\mathbf{A} = \frac{Q_{\text{enclosed}}}{\varepsilon_0} = \frac{1}{\varepsilon_0} \oint \mathbf{E} \cdot d\mathbf{A} = \frac{1}{\varepsilon_0} \cdot \frac{Q}{4\pi R^2} \]

What about inside? \[ E \cdot 4\pi r^2 = \frac{1}{\varepsilon_0} \left( \frac{3Q}{4\pi R^3} \right) \]

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inside: outside

\[ 4\pi r^2 E = \frac{Q}{\varepsilon_0} \frac{r^3}{R^3} \]

\[ E = \frac{Q}{4\pi \varepsilon_0} \frac{r^3}{R^3} \cdot \frac{1}{4\pi \varepsilon_0} = \frac{1}{3\varepsilon_0} \]

\[ \frac{Q}{4\pi \varepsilon_0} \]

Finite line of charge density \( \lambda \):

\[ E \perp \text{line, along a "radius" (cylindrical)} \]

\[ \oint \mathbf{E} \cdot d\mathbf{A} = 0 \]

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Note: extensions: spherical or infinite cylindrical shells of charge
Flat infinite plane of charge, density $\sigma$, enclosed area

$$\int \vec{E} \cdot d\vec{A} = \frac{\text{Beneath}}{\varepsilon_0}$$

$$2 \pi r^2 E = \frac{\pi r^2 \sigma}{\varepsilon_0}$$

2 ends, no flux through sides

$$E = \frac{\sigma}{\varepsilon_0}$$

But if the field goes to 0 on one side $E = 0/\varepsilon_0$

Example: Spherical shell of charge $Q$, $r = Q/(4 \pi \varepsilon_0)$

$$E = \frac{Q}{4 \pi \varepsilon_0 r^2} = \frac{4 \pi r^2 \sigma}{4 \pi \varepsilon_0 r^2} = \frac{\sigma}{\varepsilon_0}$$

As for point charge $E = \frac{Q}{4 \pi \varepsilon_0 r^2}$

We will also see this for conductors. Be careful!

Infinite planes can be confusing. Divide a plane into 2.

Is that right?

$$\int \vec{E} \cdot d\vec{A} = \frac{\text{Beneath}}{\varepsilon_0}$$

$$\frac{\sigma}{2 \varepsilon_0}$$

$$\frac{\sigma}{4 \varepsilon_0}$$

OK
1. Infinite cylinder of charge

\[ \text{charge density } \lambda \text{ coulombs/meter} \]

Find \( \vec{E}(r) \) for all \( r \).
Inside or outside?

2. A point charge \( Q \) is at the origin, \( x=y=z=0 \).
A spherical shell of charge \(-2Q\) has radius \( R \) and is centered at the origin.
Find \( \vec{E}(r) \) for all \( r \)!
"Inside" or "outside"?

3. There is a uniform, constant electric field \( \vec{E} = 4\hat{i} + 3\hat{j} + 6\hat{k} \) (mks units).
Find the flux through a vertical plane perpendicular to the \( x \)-axis, defined by \( x=5 \). (The \( xy \) plane is horizontal) of area \( 1\text{m}^2 \). Does it matter if the \( 1\text{m}^2 \) section is slid to different parts of the plane?

5. A sphere of radius \( 1\text{m} \) and charge \( 1\text{C} \) is at the origin. A point charge of \(-1\text{C} \) is at \( x=2\text{m}, y=z=0 \). What is the flux through a box of side \( 5\text{m} \) centered at the origin?
(6) A sphere of radius $R$ and charge density $p$ and centered at $z=0$ has a hollow sphere carved into it, of radius $R/2$, centered at $z=-R/2$. Find $\vec{E}$ along the $z$ axis.

Hint: superposition