

Advanced General Physics 323/324

Classical Mechanics Unit W1: The Harmonic Oscillator

Overview

This unit treats the mechanics of the harmonic oscillator, one example of which is a mass on a spring. The mathematics here has many applications in physics. Almost any system which is in stable equilibrium will undergo simple harmonic motion if displaced slightly from equilibrium. The electromagnetic field in a cavity is equivalent to an infinite collection of harmonic oscillators of different frequencies. The mathematics of AC electric circuits is essentially identical to that used in this unit.

Read: D. Kleppner and R. Kolenkow, *An Introduction to Mechanics*, Chapt. 10 - The Harmonic Oscillator

COMMENT: If you start with this unit before doing unit CM1 you may need to review the concepts of work and energy (Chap. 4). You should work through notes 10.1 and 10.2 which describe the use of complex variables in the analysis of periodic motion. The mathematical level of this discussion is not all that sophisticated and you will find this technique useful in periodic motion analysis and again in the unit AC (AC circuits) should you choose to do it.

After completing this unit you should understand:

(a) The undamped, unforced harmonic oscillator with the equation of motion

$$d^2x/dt^2 + \omega_0^2x = 0$$

, where ω_0 is the natural angular frequency of the motion. This equation has the simple solution $x(t) = A \cos(\omega_0 t + \phi)$, where A and ϕ depend on the initial conditions.

(b) The damped unforced harmonic oscillator with the equation of motion

$$d^2x/dt^2 + \gamma dx/dt + \omega_0^2x = 0,$$

where γ is the damping constant. This equation has the simple solution $x(t) = A \exp(-\gamma/2t) \cos(\omega_1 t + \phi)$, where $\omega_1^2 = \omega_0^2 - \gamma^2/4$ and A and ϕ depend on the initial conditions.

(c) The damped forced harmonic oscillator with the equation of motion

$$d^2x/dt^2 + \gamma dx/dt + \omega_0^2x = (F_0/m) \cos(\omega t),$$

where ω is the driving frequency and m is the mass of the particle. This equation has the steady-state (after a long time has passed) solution $x(t) = A \cos(\omega t + \phi)$, where A and ϕ are determined by the values of F_0, M, γ, ω and ω_0 and do *not* depend on the initial conditions .

(d) Why the solution of the damped unforced harmonic oscillator can be added to the solution in (c) to obtain the general solution for the damped forced harmonic oscillator.

Problems:

K & K Chap. 10, 1-7,9-12.

The address of this page is <http://www.physics.rutgers.edu/ugrad/323/W1.html>
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