Advanced General Physics 323/324

Electricity and Magnetism Unit E5: Maxwell's Equations

Prerequisite: E4, W1

Overview Maxwell's modification of Ampere's Law is an example of a new physical law found, not from experiment, but from carefully analyzing the inconsistency of the existing laws and finding a way to modify them so as to make them consistent. The modified laws had important consequences, which were verified by experiment. Most new physical laws are not found in this way, but when they are it is a triumph of human intellect.

Section I: Maxwell's Laws

Read: E. M. Purcell, *Electricity and Magnetism, Berkeley Physics Course, Vol. 2, 2nd Ed.* , Chapt. 9 - Maxwell's Equations and Electromagnetic Waves, Sec. 9.1-9.3.

Understand: (a) Recall Gauss' laws for the electric flux and the magnetic flux through a closed surface S surrounding a volume V:

$$\oint_S \mathbf{E} \cdot d\mathbf{S} = 4\pi \int_V \rho dV$$

$$\oint_S \mathbf{B} \cdot d\mathbf{S} = 0.$$

(b)Recall Faraday's law relating the EMF around a closed loop to the negative of the rate of change of the magnetic flux through the loop:

$$\oint \mathbf{E} \cdot d\mathbf{l} = -(1/c) \int \partial\mathbf{B}/\partial t \cdot d\mathbf{S}$$

(c) Why Ampere's law

$$\oint \mathbf{B} \cdot d\mathbf{l} = -(4\pi/c) \int \mathbf{j} \cdot d\mathbf{S},$$

relating the line integral of the magnetic field around a closed loop to the electric current through the loop, is not consistent with the above laws and therefore must be modified.

(d) How one determines the modification and finds the extra term proportional to the rate of change of the electric flux through the loop which must be added to Ampere's law:

$$\oint \mathbf{B} \cdot d\mathbf{l} = -(4\pi/c) \int \mathbf{j} \cdot d\mathbf{S} + (1/c) \int \partial \mathbf{E}/\partial t \cdot d\mathbf{S}$$

The two Gauss' laws, Faraday's law, and the modified Ampere's law are known collectively as Maxwell's equations.

Maxwell's equations above are written in their "integral forms". You will probably find it useful to look at the equivalent "differential forms" in Eqn. 15 on page 330 of Purcell. In these equations the symbols "curl" and "div" are both linear in the three partial differential operators and operate on vector fields. The curl acts like a vector cross product and operating on a vector field $\mathbf{C}(\mathbf{r})$ produces another vector field:

$$curl\mathbf{C} = \mathbf{i}(\partial C_z/\partial y - \partial C_y/\partial x + \mathbf{j}(\partial C_x/\partial z - \partial C_z/\partial x) + \mathbf{k}(\partial C_y/\partial x - \partial C_x/\partial y),$$

where $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are unit vectors in the x, y and z directions. In contrast the divergence (div) acts like a dot product and operating on a vector field produces a scalar field:

$$div\mathbf{C}(\mathbf{r}) = \partial C_x/\partial x + \partial C_y/\partial y + \partial C_z/\partial z$$

. Of the four Maxwell's equations, then, the two curl equations are vector equations while the two divergence equations are scalar equations.

Section II: Electromagnetic Waves

Read:

E. M. Purcell, *Electricity and Magnetism, Berkeley Physics Course, Vol. 2, 2nd Ed.* , Chapt. 9 - Maxwell's Equations and Electromagnetic Waves, Sec. 9.4-9.6.

Understand:

(1) For an electromagnetic wave travelling in the postitive y direction and polarized in the z direction the two fields (in CGS units) are

$$\mathbf{E} = \mathbf{k}E_0 \sin[(2\pi/\lambda)(y - ct)]$$

and

$$\mathbf{B} = \mathbf{i}E_0 \sin[(2\pi/\lambda)(y - ct)]$$

(2) The wave carries energy in the direction in which it is travelling and at the speed of light. The time average energy per area per time is $S =< E^2 > c/(4\pi) = E_0^2 c/(8\pi)$, since the time average of the square of a quantity varying sinusoidally with time is just half of its maximum value squared.

Problems: Chap. 9: 1, 2, 5, 10