CHAPTERS 1-4

VIBRATING SYSTEM - MECHANICAL SYSTEMS

- Spring
- Pendulum
- Helmholtz Resonator

Coupled Systems

WAVES - SPEED OF A WAVE

- Sound
- Water
- E&M Light

Types:
- Compressional
- Transverse

Properties

\[ k \lambda = 0 \]

Doppler shifts, interference, diffraction, reflection and refraction, standing waves.

Quantum mechanics - Waves associated with particles

* Additional topics not in textbook used for comparison
### Production of Sound by Musical Instruments

<table>
<thead>
<tr>
<th>String</th>
<th>Wind</th>
<th>Percussion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Violin</td>
<td>Vibrations of Air Column</td>
<td>Vibrations of a Drum</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>a)</th>
<th>b)</th>
<th>c)</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Diagram of string displacement" /></td>
<td><img src="image2" alt="Diagram of wind vibrations" /></td>
<td><img src="image3" alt="Diagram of percussion vibrations" /></td>
</tr>
</tbody>
</table>

2-D Version of a String

- **a**: Fundamental or 1st Harmonic
- **b**: 2nd Harmonic or 1st Overtone
- **c**: 3rd Harmonic or 2nd Overtone

Metal Bars

Stiffness Important

Tuning Forks

Organ Pipe

Edge Tone

Vortices
CHAPTER 4 - QUICK REVIEW OF ELEMENTARY PHYSICS

UNITS
- MASS - KILOGRAMS
  \[1 \text{ gram} = \frac{1}{1000} \text{ kg}\]
- LENGTH - METERS
  \[1 \text{ cm} = \frac{1}{100} \text{ m}\]  \[1 \text{ mm} = \frac{1}{1000} \text{ m}\]
- TIME - SEC
  \[1 \text{ year} = 3.15 \times 10^7 \text{ s}\]

COMBINE QUANTITIES

\[\text{SPEED} = \frac{\text{DISTANCE}}{\text{TIME}}\]
\[\text{Speed of sound in air at } 0^\circ \text{C} = 331.4 \text{ m/s}\]

\[\text{VELOCITY} \quad \vec{v} \quad \text{VECTOR} = \frac{\text{DISPLACEMENT}}{\text{TIME}}\]

\[\text{ACCELERATION} \quad \vec{a} = \frac{\Delta \vec{v}}{\Delta t} \Rightarrow \frac{\Delta \vec{v}}{\Delta t} = \frac{\Delta v}{\Delta t} = \frac{d\vec{v}}{dt} = \frac{d^2 \vec{x}}{dt^2}\]

\[\vec{F} = m\vec{a} \quad \text{Kg} \cdot \text{m/s}^2 \quad 1 \text{ Kg} \cdot \text{m/s}^2 = 1 \text{ N} \quad \text{NEWTON}\]

WORK ENERGY POWER

WORK & DIMENSION

1-D \[\text{d}W = F \text{d}x \quad \text{d}W = \vec{F} \cdot \vec{d}x\]

2-D SURFACE TENSION - USE IN DRUMS

\[\text{d}W = S_T \text{d}A \quad \text{WORK DONE TO INCREASE AREA} \quad S_T = F/\text{LENGTH}\]

3-D PRESSURE \( P \quad P = \frac{F}{\text{AREA}} \)

\[\text{d}W = P \text{d}V \quad \text{d}V = \text{VOLUME}\]
ENERGY
KE, KINETIC ENERGY \( \rightarrow J \)
UNIT
Joule \( \text{kg} \cdot \text{m}^2 / \text{s}^2 \)
\( 1 \text{J} = 1 \text{kg} \cdot \text{m}^2 / \text{s}^2 \)

MOMENTUM \( \vec{P} = m \vec{V} \)

POTENTIAL ENERGY
GRAVITY
\( \Delta \text{PE} = mgh \)

SPRING
\( F = -kx \)
\( w = \int_0^{x_f} \frac{1}{2} k x^2 \, dx \)

POWER
\( \text{ENERGY/TIME} = \text{Joule/s} \)
\( 1 \text{J/s} = 1 \text{Watt} \)

MASS COMBINED WITH LENGTHS...
M/L, MASS/LENGTH = WIRES
M/AREA = MEMBRANES
M/VOLUME = DENSITY \( \rho \)
\( \rho_{\text{water}} = 1 \text{gram/cm}^3 \)
\( \rho_{\text{air}} = 1.29 \text{ kg/m}^3 \)

PRESSURE \( P = F/A \) SCALAR
\( 1 \text{ ATMOSPHERE} = 1.01 \times 10^5 \text{ N/m}^2 \)
\( = 1.01 \times 10^5 \text{ kg/m/s}^2 \)
Position, Velocity, Acceleration

Initially $x=0$ compressed.

$x = A \cos 2\pi ft$

$s = \frac{1}{2} \frac{A}{T_0}$

$T_0$ period

$\frac{dx}{dt} = -2\pi f A \sin 2\pi ft$

$\frac{d^2x}{dt^2} = -2\pi f A \cos 2\pi ft$

$m = 0$

$\frac{d^2x}{dt^2} = -m \frac{d^2x}{dt^2}$

Position

Moving fastest at equilibrium position

Slope of $x$ curve $m$

Slope of $v$ curve $m$

$T_0/4$

$T_0$

$3T_0/4$

$T_0$

$T_0$

$a = \frac{d^2x}{dt^2} = -\left(2\pi f\right)^2 A \cos 2\pi ft = -\omega^2 x$

$\omega = 2\pi f$

$ma = -m \omega^2 x = -kx$

$kx = \sqrt{k/m}$
### Properties of Waves

<table>
<thead>
<tr>
<th>Sound</th>
<th>E&amp;M Light</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Type</strong></td>
<td><strong>Electric &amp; Magnetic</strong></td>
</tr>
<tr>
<td>Pressure Wave or Density</td>
<td><strong>Fields Vibrate</strong></td>
</tr>
<tr>
<td>Wave</td>
<td></td>
</tr>
</tbody>
</table>

> PROPAGATION: **Longitudinal**

**Sound in Solids**
- Is both transverse & longitudinal
- Example: earthquakes

**Speed of Sound**
- Does not depend on \( \lambda \) or \( f \)

**Note** \( \nu = \lambda f \)
- \( \nu \) fixed

**\( \lambda = VT \)**
- \( T = \frac{1}{f} \) period

**Source produces 1\( \lambda \) per period.**

**\( V_s = 344 \text{ m/s} \approx 3 \times 10^2 \)**

**\( c = 3 \times 10^8 \text{ m/s} \)**

**\( c = 10^6 \times V_s \text{ million times faster} \)**
## WAVES

<table>
<thead>
<tr>
<th><strong>SOUND</strong></th>
<th><strong>LIGHT</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>HEARING</strong></td>
<td><strong>SEEING</strong></td>
</tr>
<tr>
<td>$f = 20 \text{ Hz to } 20,000 \text{ Hz}$</td>
<td>$f = 4.2 \times 10^{14} \text{ to } 7.4 \times 10^{14} \text{ Hz}$</td>
</tr>
<tr>
<td>- FACTOR OF 1000</td>
<td>- FACTOR LESS THAN 2</td>
</tr>
<tr>
<td>$1 \text{ Hz} = 1 \text{ cycle/s}$</td>
<td>$\lambda = \frac{3 \times 10^8}{2 \times 10^{14}} = 1.5 \times 10^{-6} \text{ m}$</td>
</tr>
<tr>
<td>$\lambda = 33.4 \text{ m}$</td>
<td>$\lambda = 10^3 \text{ m}$</td>
</tr>
<tr>
<td>$f = 33.4 \text{ Hz}$</td>
<td>$\lambda = 10^{-3} \text{ m}$</td>
</tr>
<tr>
<td>$\lambda = 3 \times 10^8 \text{ Hz}$</td>
<td>$f = 3340 \text{ Hz}$</td>
</tr>
<tr>
<td>$f = 33 \text{ m}$</td>
<td>$\lambda = 0.1 \text{ m}$</td>
</tr>
<tr>
<td>$\lambda = 3 \times 10^8 \text{ Hz}$</td>
<td>$f = 3340 \text{ Hz}$</td>
</tr>
</tbody>
</table>

### SOUND

<table>
<thead>
<tr>
<th><strong>PIANO:</strong> LOWEST NOTE</th>
<th><strong>ORCHESTRA'S TUNE AT</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_0 = 27.5 \text{ Hz}$</td>
<td>$A_4 = 440 \text{ Hz}$</td>
</tr>
<tr>
<td>$C_4 = 418.6 \text{ Hz}$</td>
<td>$\text{MIDDLE } C_4 = 262 \text{ Hz}$</td>
</tr>
</tbody>
</table>

### LIGHT

<table>
<thead>
<tr>
<th><strong>COLOR</strong></th>
<th><strong>WAVELENGTH</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>RED</strong></td>
<td>660 - 700 nm</td>
</tr>
<tr>
<td><strong>ORANGE</strong></td>
<td>597 - 660 nm</td>
</tr>
<tr>
<td><strong>YELLOW</strong></td>
<td>577 - 597 nm</td>
</tr>
<tr>
<td><strong>GREEN</strong></td>
<td>492 - 577 nm</td>
</tr>
<tr>
<td><strong>BLUE</strong></td>
<td>455 - 492 nm</td>
</tr>
<tr>
<td><strong>INDIGO</strong></td>
<td>390 - 455 nm</td>
</tr>
<tr>
<td><strong>VIOLET</strong></td>
<td>350 - 390 nm</td>
</tr>
</tbody>
</table>
Properties of Waves

Using Dimensional Analysis to Find Speed of Sound

Speed on a String

Put string under tension - $T = F \cdot \text{kg} \cdot \text{m/s}^2$

Properties of a String

Mass $M$\hspace{1cm}$\text{kg}$
Length $L$\hspace{1cm}$\text{m}$

Why must involve all three
Can't be just the mass
Or just the length alone

$u \sim (T)^{1/2}$ to some power

Why must involve $M/L$
For example if it involved just $M$ or $L$ alone then if you cut the string in half you would get a different speed

$T$ is in numerator $T \rightarrow 0 \quad u \rightarrow 0$

$T$ has $\text{kg} \Rightarrow \frac{T}{M/L}$

$T$ has $\text{s}/\text{sec}$ $\Rightarrow$ this cancels $\text{kg}$

$u = \sqrt{\frac{T}{M}}$ units $\sqrt{\frac{\text{kg} \cdot \text{m}}{\text{s}^2}} = \frac{\text{m}}{\text{sec}}$

So up to some dimensionless constant
This is the right answer
SPEED ON A MEMBRANE - 2D SYSTEM

SURFACE TENSION \( T_s \) \( dW = T_s \text{d} \text{Area} \)

\[ dW = \frac{K_g \cdot m \cdot m}{s^2} = T_s \cdot m^2 \cdot \frac{T}{s} = \frac{K_g}{s^2} = \frac{F}{l^2} \]

MEMBRANE HAS A MASS \( M \)

AREA \( A \)

\( V \) MUST INVOLVE \( M/A \)

\[ T_s \sim \frac{1}{s^2} \Rightarrow V \propto \sqrt{\frac{T_s}{M/A}} \]

SPEED IN A GAS - 3D PROPERTIES

1D 2D 3D

\( T \ (F) \ T_s \ (F/k) \ P : \text{PRESSURE} \ (F/l^2 = F/A) \)

\( M/L \ M/A \ M/N = \rho \)

\[ dW = T \text{d}x \quad dW = T_s \text{d}A \quad dW = P \text{d}V \]

\[ V \propto \sqrt{\frac{P}{\rho}} \quad \text{DIMENSIONLESS} \quad \sqrt{\frac{\gamma \rho}{\rho}} \]

\[ \gamma = 1.4 \quad \left( \frac{C_p}{C_v} \quad \text{SPECIFIC HEAT CONSTANT} \ \rho \right) \]

Actually,

\[ V = \sqrt{\frac{B}{\rho}} \quad B : \text{BULK MODULUS} \]

\[ B = -V \left( \frac{\partial P}{\partial V} \right) \quad dQ = 0 \quad \text{NO HEAT} \]

\[ \Rightarrow \text{SENTROPY} = 0 \]
WHAT HAPPENS WHEN \( d \ll \lambda \)

\[
\lambda \rightarrow \lambda
\]

\[
\text{DEEP } d \gg \lambda \quad v = \sqrt{\frac{g\lambda}{2\pi}} = \sqrt{\frac{g}{2\pi}} \quad z = \frac{2\pi}{\lambda} \quad \text{WAVE NUMBER}
\]

\( d \) SMALL COMPARED TO \( \lambda \) - BUT WAVE IS GRAVITY DRIVEN - ALSO NEED \( g \) FOR \( \text{sec}^2 \).

\[
v = \sqrt{gd} \quad \text{SHALLOW } \quad d \ll \sqrt{d}.
\]

\[
v_{\text{DEEP}} = \sqrt{\frac{g\lambda}{2\pi}} \quad v_{\text{SHALLOW}} = \sqrt{gd}
\]

\[
\frac{v_{\text{SHALLOW}}}{v_{\text{DEEP}}} = \sqrt{\frac{2\pi}{\lambda}} \quad \text{BUT USE SHALLOW WHEN } d \ll \lambda
\]

\[
\frac{v_{\text{SHALLOW}}}{v_{\text{DEEP}}} < 1 \quad \text{WHEN } 2\pi d < \lambda
\]

\[
\text{WHEN } d/\lambda < 1/10 \quad \text{SHALLOW LIMIT}
\]

WAVE SLOWS DOWN AS IT APPROACHES SHORE.

\[
v = \sqrt{\frac{g\lambda}{2\pi}} \tan h \frac{4\pi d}{\lambda}
\]

\[
\tan h x \rightarrow x \quad \text{SMALL } x
\]

\[
\rightarrow 1 \quad \text{LARGE } x
\]

\[
x = \frac{2\pi d}{\lambda} \quad \text{SHALLOW } \lambda \ll d \quad \text{X SMALL}
\]

\[
x \ll \lambda \quad \text{DEEP } d \gg \lambda \quad \text{X LARGE}
\]
ENERGY STORED IN A MOVING WAVE

1. FACTOR 1 - AMPLITUDE SQUARE - JUST LIKE A SPRING - $A^2$ KINETIC ENERGY $(kA)^2$

2. FACTOR 2 - FOR EVERY ENERGY TRANSFERRED MULTIPLY BY SPEED OF WAVE $U$

3. THE PRODUCT $U \cdot A^2$ STAYS CONSTANT IF NO LOSSES ARE PRESENT

LET $A = k$ HEIGHT OF THE WAVE

$U \cdot A^2 = \text{constant}$

SHALLOW WAVES $U \sim \sqrt{gd}$, $d = \text{depth}$

$\sqrt{gd} \cdot A^2 = C$, $k^2 \sim \frac{C}{\sqrt{gd}}$, $h \sim \frac{1}{d^{1/4}}$

HEIGHT OF WAVE GROWS AS IT GETS CLOSER TO SHORE

AT CERTAIN HEIGHT WAVE BREAKS

\[ \begin{align*}
\text{Friction from ocean} \\
\text{Floor in surf zone} \\
\text{Velocity profile of water}
\end{align*} \]
ADDITIONAL NOTES ON LAB

LAB 1 MENTIONS SCALES

CHROMATIC SCALE IS A 12 NOTE SCALE.

ON THE 13TH KEY YOU ARE BACK TO THE SAME
NOTE BUT AN OCTAVE HIGHER. A OCTAVE
IS A FACTOR OF 2, HIGHER IN FREQUENCY.

HERE IS AN OCTAVE ON A PIANO KEYBOARD
STARTING WITH C

<table>
<thead>
<tr>
<th>SHARPS</th>
</tr>
</thead>
<tbody>
<tr>
<td>C#</td>
</tr>
<tr>
<td>D#</td>
</tr>
<tr>
<td>F#</td>
</tr>
<tr>
<td>G#</td>
</tr>
<tr>
<td>A#</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>FLATS</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
</tr>
<tr>
<td>E</td>
</tr>
<tr>
<td>G</td>
</tr>
<tr>
<td>A</td>
</tr>
<tr>
<td>B</td>
</tr>
</tbody>
</table>

PENTATONIC SCALE - IS A 5 NOTE SUBSET
ON THE CHROMATIC SCALE - ON THE PIANO
THE BLACK KEYS FORM A PENTATONIC SCALE
THE WHITE KEYS FORM A DIATONIC SCALE

JUST INTONATION TUNED - RATIO OF SMALL NUMBERS

<table>
<thead>
<tr>
<th>DIATONIC PENTATONIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
</tr>
<tr>
<td>C/D♭</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>MINOR</td>
</tr>
<tr>
<td>2ND</td>
</tr>
</tbody>
</table>
ADDITIONAL NOTES ON LAB 1

ANOTHER SCALE IS THE EQUAL TEMPERED SCALE
THIS SCALE USES POWERS OF $2^{1/12} = 1.05946$
TO GO UP FROM C $\rightarrow$ OCTAVE C
THE RATIO OF ANY TWO ADJACENT KEYS IS $2^{1/12}$
I.E. EQUAL RATIOS. IT IS THEREFORE AN
EXPONENTIALLY INCREASING SCALE

\[
\begin{align*}
C & : 2^{0} \\
C^\# / C & : 2^{1/12} \\
D / C & : (2^{1/12})^2 = 2^{1/6} \\
\# / C & : (2^{1/12})^3 = 2^{1/4} \\
E / C & : (2^{1/12})^4 = 2^{1/3} \\
F / C & : (2^{1/12})^5 = 2^{5/12} \\
\# / C & : (2^{1/12})^6 = \sqrt{2} \\
G / C & : (2^{1/12})^7 = 2^{7/12} \\
\# / C & : 2^{9/12} = \frac{3}{2} \\
A / C & : 2^{9/12} = 2^{3/4} \\
A^{\#} / C & : 2^{10/12} = 2^{5/6} \\
B / C & : 2^{11/12} \\
C^\prime / C & : 2^{13/12} = 2^0 = 2
\end{align*}
\]

\[
(2^{1/12})^m \quad \text{WRITE} \quad 2 = e^{\ln 2} \\
(2^{1/12})^m = 2^{m/12} = (e^{\ln 2^{1/12}})^m \Rightarrow \text{EXPONENTIALLY INCREASING SCALE}
\]

LOWEST NOTE ON A PIANO IS $A_0 = 27.5 \text{ Hz}$. SUBSCRIPT 0 = OCTAVE
HIGHEST KEY IS KEY 88. - $C_8 = 4186.3 \text{ Hz}$
\[
\text{N}^{\text{th}} \text{ KEY ABOVE } A_0 \text{ HAS FREQUENCY } (27.5)^n \text{ Hz } \times 2^{n/12}
\]

$C_1$ IS 1ST OCTAVE C = 32.76 Hz = $2^{3/12} \times A_0 = 2^{1/4}$

$C_8 = 2^7 C_1 = 128 \times 37.1$
<table>
<thead>
<tr>
<th>Groups of 12</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>FIRST OCTAVE</td>
<td>A₀</td>
<td>A♯</td>
<td>B₀</td>
<td>C₁</td>
<td>C♯</td>
<td>D₀</td>
<td>D♯</td>
<td>E₁</td>
<td>F₁</td>
<td>F♯</td>
<td>G₁</td>
<td>G♯</td>
</tr>
<tr>
<td>C₇ →</td>
<td>27.5</td>
<td>30.9</td>
<td>34.6</td>
<td>37.6</td>
<td>41.2</td>
<td>43.7</td>
<td>49</td>
<td>51.9</td>
<td>55</td>
<td>61.7</td>
<td>66.4</td>
<td>74.0</td>
</tr>
<tr>
<td>2ND OCTAVE</td>
<td>A₁</td>
<td>A♯</td>
<td>B₁</td>
<td>C₂</td>
<td>C♯</td>
<td>D₂</td>
<td>D♯</td>
<td>E₂</td>
<td>F₂</td>
<td>F♯</td>
<td>G₂</td>
<td>G♯</td>
</tr>
<tr>
<td>C₉ →</td>
<td>55</td>
<td>61.7</td>
<td>66.4</td>
<td>74</td>
<td>82.4</td>
<td>93</td>
<td>102</td>
<td>110</td>
<td>116.5</td>
<td>123.4</td>
<td>131.2</td>
<td>140.8</td>
</tr>
<tr>
<td>3RD OCTAVE</td>
<td>A₂</td>
<td>A♯</td>
<td>B₂</td>
<td>C₃</td>
<td>C♯</td>
<td>D₃</td>
<td>D♯</td>
<td>E₃</td>
<td>F₃</td>
<td>F♯</td>
<td>G₃</td>
<td>G♯</td>
</tr>
<tr>
<td>C₁ →</td>
<td>110</td>
<td>116.5</td>
<td>123.4</td>
<td>131.2</td>
<td>140.8</td>
<td>150.8</td>
<td>160.2</td>
<td>169.9</td>
<td>179.6</td>
<td>190.0</td>
<td>200.7</td>
<td>212</td>
</tr>
<tr>
<td>4TH OCTAVE</td>
<td>A₃</td>
<td>A♯</td>
<td>B₃</td>
<td>C₄</td>
<td>C♯</td>
<td>D₄</td>
<td>D♯</td>
<td>E₄</td>
<td>F₄</td>
<td>F♯</td>
<td>G₄</td>
<td>G♯</td>
</tr>
<tr>
<td>C₂ →</td>
<td>220</td>
<td>233</td>
<td>246.9</td>
<td>261.6</td>
<td>277.2</td>
<td>293.6</td>
<td>311.1</td>
<td>329.6</td>
<td>349.2</td>
<td>370.0</td>
<td>392.0</td>
<td>415.3</td>
</tr>
<tr>
<td>5TH OCTAVE</td>
<td>A₄</td>
<td>A♯</td>
<td>B₄</td>
<td>C₅</td>
<td>C♯</td>
<td>D₅</td>
<td>D♯</td>
<td>E₅</td>
<td>F₅</td>
<td>F♯</td>
<td>G₅</td>
<td>G♯</td>
</tr>
<tr>
<td>C₄ →</td>
<td>440</td>
<td>466.1</td>
<td>493.9</td>
<td>523.2</td>
<td>554.4</td>
<td>587.2</td>
<td>622.2</td>
<td>659.2</td>
<td>698.4</td>
<td>740.0</td>
<td>784.0</td>
<td>830.6</td>
</tr>
<tr>
<td>6TH OCTAVE</td>
<td>A₅</td>
<td>A♯</td>
<td>B₅</td>
<td>C₆</td>
<td>C♯</td>
<td>D₆</td>
<td>D♯</td>
<td>E₆</td>
<td>F₆</td>
<td>F♯</td>
<td>G₆</td>
<td>G♯</td>
</tr>
<tr>
<td>C₅ →</td>
<td>880</td>
<td>925</td>
<td>976</td>
<td>1027</td>
<td>1080</td>
<td>1134</td>
<td>1190</td>
<td>1252</td>
<td>1318</td>
<td>1388</td>
<td>1463</td>
<td>1544</td>
</tr>
<tr>
<td>7TH OCTAVE</td>
<td>A₆</td>
<td>A♯</td>
<td>B₆</td>
<td>C₇</td>
<td>C♯</td>
<td>D₇</td>
<td>D♯</td>
<td>E₇</td>
<td>F₇</td>
<td>F♯</td>
<td>G₇</td>
<td>G♯</td>
</tr>
<tr>
<td>C₆ →</td>
<td>1760</td>
<td>1835</td>
<td>1918</td>
<td>2007</td>
<td>2097</td>
<td>2188</td>
<td>2283</td>
<td>2383</td>
<td>2485</td>
<td>2592</td>
<td>2704</td>
<td>2820</td>
</tr>
</tbody>
</table>

- **F 参数**: 27
- **F 参数**: 128
- **High Pitch = High Frequency**
- **Semitone = One Note Above A Note Raising F by 12(2 increases pitch by one semitone.**
BASIC SHAPE

FOR A VIOLIN
GDAE

PEG

FINGERBOARD - NECK

70D PLATE

SOUND HOLE

BRIDGE

BACK PLATE

INCREASING SIZE

4/4 4/4 4/4

LARGE BASS - F A D G C
E1 A1 D2 G2
41.2 55 73.4 98

SMALL BASS A D G C
A1 D2 G2 C3
55 73.4 98 130.8

NEW CELLO CGDA
C2 G2 D3 A3
65.4 98 146.8 220

TENOR VIOLIN GDAE
G2 D3 A3 E4
98 146.8 220 329.5

VERTICAL VIOLA CGDA
C3 G3 D4 A4
131 196 293 440

VIOLIN GDAE
G3 D4 A4 E5
196 293 440 659

SOPRANO VIOLIN CGDA
C4 G4 D5 A5
262 392 588 880

TREBLE VIOLIN GDAE
G4 D5 A5 E6
392 588 880 1318

CGDA GROUP: NEW CELLO + 1 OCTAVE = VERTICAL VIOLA
VERTICAL VIOLA + 1 OCTAVE = SOPRANO VIOLIN
GDAE GROUP: TENOR + 1 OCTAVE = VIOLIN
VIOLIN + 1 OCTAVE = TREBLE
Music is clock counting

\[ C, C^\#, D, D^\#, E, E^\#, F, F^\#, G, G^\#, A, A^\#, B, \text{ reset clock} \]

\[ C_2, C_2^\#, \ldots \]

\[ B_2 \]

\[ C_3, C_3^\#, \ldots \]

\[ B_3 \]

Pitch Helix (called spiral)

Going up the spiral is going up in octaves

Going around a loop goes through the various f's of the scale \( C_n, C_n^\#, D_n, D_n^\#, E_n, E_n^\#, F_n, F_n^\#, G_n, G_n^\#, A_n, A_n^\#, B_n \)

\[ n = 1, 2, 3, \ldots \]

\[ C_2 = 2^n C_1 \]

Pitch Height (octaves)

Pitch Chroma

Most famous helix in nature
**Pitch Spiral - Equal Tempered Scale**

Write \( r = r_0 e^{\alpha \theta} \) \( \alpha \) is a real number.

\( \theta ; 0 \leq \theta \leq 2\pi \) for 1st revolution (1st octave)

\( \theta ; 2\pi \leq \theta \leq 4\pi \) for 2nd revolution (2nd octave)

Let \( \theta = 0 \) for \( C \), \( r = r_0 e^0 = r_0 \) frequency of \( C \)

At \( \theta = 2\pi \) have \( C_2 \) with \( f = 2f \) (for \( C \))

\[ r_{C_2} = \frac{r_0 e^{\alpha \cdot 2\pi}}{r_0} = \frac{r_0}{r_0} e^{\alpha \cdot 2\pi} = e^{\alpha \cdot 2\pi} \]

Take \( e^{\alpha \cdot 2\pi} = e^{2\pi \alpha} \) \( \alpha = \frac{\ln 2}{2\pi} \)

**Pitch Spiral** \( r = r_0 e^{\frac{\theta}{(2\pi \alpha)}} \)

Check when \( \theta = 4\pi \) should have \( C_2 = 2 \cdot 2^\alpha = 4 \cdot 2^\alpha \)

\[ r = r_0 e^{\frac{1}{(2\pi \alpha)(4\pi)}} = r_0 e^{\frac{2\pi \ln 2}{4\pi \alpha}} = r_0 e^{\frac{\ln 2}{2\pi \alpha}} = 2 \cdot 2^{\alpha} = 4 \cdot 2^\alpha \]

Other spirals
- Spiral built on golden mean.
- Spirals in nature: Cornu spiral in electron diffraction.