Pipes Versus Strings.

Sound in pipes - longitudinal waves in a string - transverse waves.

\[ v = 331 \, \text{m/s} + 0.6 \, T(\degree \text{C}) \]

\[ v = \sqrt{\frac{k}{\rho}} \]

\[ B = -\sqrt{\frac{\rho}{\gamma \Delta p}} \]

\[ \gamma = \frac{c_p}{c_v} \]

\[ \gamma_0 = \frac{c_p}{c_v} < 3/3, \text{monatomic} \]

\[ 7/5, \text{diatomic} \]

\[ 5/3 \rightarrow 7/5 \text{ come from rotations.} \]

\[ v = \sqrt{\frac{k}{\rho}} \]

\[ k = 1.38 \times 10^{-23} \, \text{J/} \text{K} \]

\[ \lambda f = v, \quad f = v/\lambda \]

Open at both ends (flute - piccolo - small flute).

\[ \lambda = 2L \]

Open at one end (clarinet - organ pipes - typical).

\[ n\lambda = 2L \]

Closed at other end (saxophone - conical).

\[ (2n-1)\lambda = 4L \]

\[ \lambda f = v, \quad f = v/\lambda \]

Node

Open anti-node

Closed anti-node

\[ 2\lambda = 4L \]

\[ 3\lambda = 4L \]
Pipes vs. Strings

Sound in pipes.

Finding $\lambda$.

**For Displacement of Air Molecules:**
- Open ends are anti-nodes - like free end of a string.
- Closed ends are node - atoms/molecules can't move.
- Closed ends are like fixed ends of a string.

**Displacement**

<table>
<thead>
<tr>
<th>Open</th>
<th>Closed</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda = 4L$</td>
<td>$3\lambda = 4L$</td>
</tr>
<tr>
<td>$n = 1, 2, 3, \ldots$</td>
<td></td>
</tr>
</tbody>
</table>

| $s = 0$ | $s = 2\lambda/4L$ | $s = 3\lambda/4L$ | $s = 5\lambda/4L$ |

Frequencies:
- Fundamental
- 1st Harmonic
- 3rd Harmonic
- 5th Harmonic

Even harmonics missing.

| $n = 1, 2, 3, \ldots$ |

Pressure - at open ends, put node - pressure goes to atmospheric pressure - sound is a $\Delta P$ - pressure varying above/below atmospheric.

Closed end put an anti-node - density builds up $\Delta P$.

Like a string.

Fixed at both ends.

At低声 by 60%.
PRESSURE IN CONICAL PIPE COMPARED TO CYLINDRICAL
OPEN END - NODE; CLOSED END - ANTI-NODE

\[ x = 0 \quad x = L \]
\[ \Delta p \sim \text{const.} \]
\[ \Delta p = 0 \quad x = 0 \]
\[ \Delta p = 0 \quad x = L \]
\[ \Rightarrow \frac{kL}{L} = \pi, \frac{2\pi}{L} \]
| \[ k = \frac{n\pi}{L} \quad n = 1, 2, \ldots \]
| \[ \lambda = \frac{2\pi}{k} \quad \lambda = \frac{2\pi L}{k} \]
| \[ n\lambda = 2L \quad (2n-1)\lambda = 4L \]

\[ \text{CONICAL} \]
\[ \Delta p \sim \frac{\text{const.} \lambda}{k} = 0 \text{ AT } \]
\[ \Rightarrow \frac{kL}{L} = \pi, \frac{2\pi}{L} \]
| \[ k = \frac{n\pi}{L} \]

\[ \text{CONICAL INSTRUMENTS: OBOE, BASSOON, ENGLISH HORN.} \]

\[ \text{RECALL INTENSITY OF SUNLIGHT FALLS AS } \frac{1}{r^2} \text{ SPHERICALLY OUTGOING} \]
\[ E_{\text{rad}} \sim \frac{1}{r} \quad I \sim \frac{E_{\text{rad}}}{\lambda^2} \]
\[ \text{AMOUNT OF ENERGY CROSSING SPHERE } \sim \int 4\pi r^2 \text{ d}E_{\text{rad}} \]
HARMONIC SPECTRUM:

Piccolo:
- Has 1/2 length of flute
- Plays at 2 x fundamental of flute

Flute:
- Has strong 2nd harmonic

Clarinet:
- Cylindrical bell
- Even harmonics
  - Weak

Saxophone:
- Conical bore
- Acts more like open-open-even
- Harmonics have strong relative strength
Both Open

\[ \lambda = \frac{2\pi}{2} \]

One Side Open

\[ \lambda = \frac{\pi}{2} \]

Cone

\[ \lambda = \frac{\pi}{2} \]

\[ \lambda = \frac{\pi}{2} \]

\[ \lambda = \frac{\pi}{2} \]

Looks like \( \lambda/4 \) but is really \( \lambda/2 \)

\[ \lambda + \lambda/2 = 3\lambda/2 \]

\[ \lambda + \lambda = 2\lambda \]

\[ \lambda/4 \]

\[ \lambda/2 \]

\[ \lambda/2 \]

\[ \lambda/2 \]

All equally spaced heights

\[ \text{REAL } \lambda/2 \]

Oboe \leftrightarrow \text{Flute}

All harmonics

Picture for oboe is misleading.

The first node to first anti-node looks like \( \lambda/4 \) but is really \( \lambda/2 \)

\[ \sin \frac{kr}{r} \to 1 \text{ as } r \to 0. \]

\[ \sin \frac{kr}{r} \to 0 \text{ as } r \to 0. \]

Need \( \frac{1}{r} \) for outgoing wave to keep power.

\[ \text{Power } \sim \left( \frac{4\pi}{r^2} \right) \sim \frac{1}{2} \text{ the same when } \times \text{ by } 4\pi. \]
**Speed of Sound in Metals**

**Stress-Strain Relations**

Young's Modulus $Y$ (Elastic Modulus, Tensile Modulus)

![Diagram of Stress-Strain Relationship]

\[ \frac{F}{A} = \frac{\Delta l}{l} \]

\[ F = Y \frac{\Delta l}{l} \]

- Why Elasticity depends on $\frac{\Delta l}{l}$

For $2\Delta l$ get $2AL$.

\[ 2\Delta l = \Delta l \]

\[ \frac{2\Delta l}{\Delta l} = 2 \]

\[ \frac{\Delta l}{\Delta l} = 1 \]

Sprink Constant $\sim$ # of coils
SPEED OF SOUND IN METALS.

2. WHY \( F/A \) \( = \frac{k}{l} \)

STACK TWO BLOCKS

\[ \frac{F}{A} \rightarrow \frac{F}{A} \rightarrow \frac{F}{A} \]

\[ \Delta x_1 = \frac{F}{2k} \]

\( \Delta x_1 = \frac{\Delta x}{2} \) STIFFNESS \( \frac{1}{2} \) AS MUCH FOR THE SAME FORCE.

\( \Delta x_2 = \) TO GET \( \Delta x = 2\Delta x \) NEED \( 2F \).

\[ V = I R_0 \]

STRESS

\[ R_{jc} = \frac{\partial l}{A} = \frac{l}{\partial A} \]

\[ V = \frac{I R}{A} \]

CURRENT DENSITY \( \text{AREA} \)

\[ y = E \text{ (ELECTRIC FIELD)} \]

\[ E = \frac{1}{C} \quad J = 0 \cdot E \]
SPEED OF SOUND IN METALS

LONGITUDINAL WAVES -

BEFORE G A B.

\[ \sigma = \sqrt{\frac{B}{\rho}} \]

\[ u = \sqrt{\frac{Y}{\rho}} \]

CHECK UNITS

\[ \frac{E}{A} = \frac{Y AL}{l} \]

\[ Y = \frac{F \ell}{A \Delta \ell} \]

\[ B = -\frac{1}{V} \frac{\partial V}{\partial \sigma} = \gamma \rho \] NO UNIT.

\[ \text{PRESSURE} \quad \text{KG.M/S}^2 \quad \text{M}^2 \quad = \frac{\text{KG}}{\text{M.S}}. \]

\[ Y_{\text{Aluminum}} = 69 \times 10^9 \left( \frac{\text{Kg}}{\text{M.S}^2} = \text{Pascals} \right) \quad \rho = 2.7 \text{ gram/cm}^3. \]

\[ Y_{\text{Steel}} = 200 \times 10^9. \quad \rho = 8 \times 10^3 \left( 10^{-2} \frac{\text{Kg}}{\text{M.O}^3} \right). \]

\[ Y_{\text{Diamond}} = 1200 \times 10^9. \quad \rho = 3.5 \times 10^3 \left( 10^{-3} \frac{\text{Kg}}{\text{M.O}^3} \right). \]

\[ Y_{\text{Glass}} = 50 - 90 \times 10^9. \quad \rho = 2.4 - 2.8 \times 10^3. \]

<table>
<thead>
<tr>
<th>SPEED OF SOUND U</th>
<th>ALUMINUM</th>
</tr>
</thead>
<tbody>
<tr>
<td>METAL ( U ) ( \text{(M/S)} )</td>
<td>( \sqrt{\frac{69 \times 10^9}{2.7 \times 10^3}} = 5.05 \times 10^3 )</td>
</tr>
<tr>
<td>ALUMINUM ( 5050 )</td>
<td>( \sqrt{\frac{69 \times 10^9}{2.7 \times 10^3}} = 5.05 \times 10^3 )</td>
</tr>
<tr>
<td>STEEL ( 5000 )</td>
<td>( 5 )</td>
</tr>
<tr>
<td>DIAMOND</td>
<td>( \rho_{\text{Diamond}} = 3.5 \text{ g/cm}^3. )</td>
</tr>
<tr>
<td>GLASS</td>
<td>( \rho_{\text{Glass}} = 8 \text{ g/cm}^3. )</td>
</tr>
</tbody>
</table>

DENSITY OF STEEL \( \sim 8 \text{ grams/cm}^3 \)

\[ Y_{\text{Steel}} = 200 \times 10^9. \]

\[ \sqrt{\frac{200 \times 10^9}{8 \times 10^3}} = \sqrt{25} \times 10^3 \]
**Shear, Bending, Torsional Forces**

**Besides elongation & the associated Young's Elastic Modulus, there is shear modulus.**

**Shear in solids.**

\[ \frac{F}{A} = \frac{Y}{s} \frac{dL}{L} = Y \epsilon \]

\[ \gamma = G \text{ (also used)} \]

**Shear in fluids.**

Fluids do not support shear - they flow.

\[ \frac{F}{A} = \frac{\eta}{\ell} \frac{dV}{dL} \]

\[ \eta = \text{viscosity} \]

\[ \eta \approx 15 \times 10^{-6} \text{ Ns/m}^2 \text{ air} \]

\[ \eta \approx 10^{-3} \text{ Ns/m}^2 \text{ water} \]

**Torsional.**

\[ \Delta L = R \phi \]

\[ \phi \text{ inside} \]

Laminar flow

Turbulent flow
TORSIONS - A PICE IN VIOLIN BOWING - BOWING

TORQUE VERSUS $\phi$

ALSO

TWISTS
WIRES

\[ \tau = \frac{F \cdot \Delta L}{L \cdot \phi} \]

\[ \Delta L = L \cdot \phi = R_0 \cdot \phi \]

\[ \tau = R_0 \cdot \frac{F}{A} = R_0 \cdot \frac{Y \cdot \Delta L}{A} \]

\[ \tau = \frac{Y \cdot \Delta L}{A} \cdot \frac{R_0}{L} \]

\[ \tau = \frac{Y \cdot \Delta L}{A} \cdot \frac{R_0^2}{L} \]

\[ \tau = \frac{Y \cdot \pi \cdot R_0^4}{L} \cdot \frac{\phi}{L} \]

EASIER TO TWIST A THIN WIRE - $R_0^4$ FACTOR

"""" """" LONG WIRE $1/L$ FACTOR

NEXT: USE $\tau$ TO CALCULATE ROTATIONAL FREQUENCY

\[ \tau = I \cdot \alpha = I \cdot \frac{d^2 \phi}{dt^2} \]
\[ T = \frac{\pi R_0^4}{2L} Y_s \phi \]

**HARMONIC MOTION**

\[ T = I \alpha \quad I = \text{MOMENT OF INERTIA} \]
\[ \alpha = \text{ANGULAR ACCELERATION} \]

\[ I \frac{d^2 \phi}{dt^2} = -k_s Y_s \phi \]

\[ Y_s \text{ intrinsic property of material} \]

\[ k_s \text{ contains } R_0, L \]

\[ k_s \text{ restoring property of object} \]

\[ \frac{d^2 \phi}{dt^2} = -k_s Y_s \phi \]

\[ \phi = \phi_0 \cos \omega t \quad \phi_0 \text{ initial } \phi \text{ of displacement} \]

\[ \frac{d \phi}{dt} = -\phi_0 \omega \sin \omega t \]

\[ \frac{d^2 \phi}{dt^2} = -\phi_0 \omega^2 \cos \omega t \]

\[ -\phi_0 \omega^2 \cos \omega t = \frac{k_s Y_s \phi_0 \cos \omega t}{I} \]

\[ \omega = \sqrt{\frac{k_s Y_s}{I}} = \sqrt{\frac{\pi R_0^4}{2L}} \frac{Y_s}{L} = \sqrt{\frac{R_0^2 \pi Y_s}{2L M_0}} \]

**CHECK UNITS**

\[ Y_s = \frac{F}{\Delta \phi} = \frac{k_s m}{\Delta \phi} \alpha \sqrt{\frac{m^2}{m^2 K_s m \frac{m}{s^2}} = \frac{1}{s}} \]
EXAMPLES OF SHM.

\[ T \approx mg \]
\[ T \cos \theta - mg = 0 \quad T \approx mg \]
\[ T \sin \theta = ma \]
\[ \sin \theta = \frac{x}{L} \]
\[ \sin \theta = \frac{-x}{L} \]
\[ -mg \frac{x}{L} - ma \frac{d^2 x}{dt^2} \]
\[ k = \frac{mg}{L} \quad \frac{1}{\ell/m} = \frac{d^2}{d^2} 

\[ \frac{d^2 x}{dt^2} = \frac{kx}{m} \]

\[ \omega^2 = \frac{g}{\ell} + \frac{k}{m} \]
Coupled Systems (p. 1)

\[ T \cos \theta = mg \]
\[ \cos \theta = 1 \]
\[ T = mg \]

\[ m \frac{d^2x_1}{dt^2} = -T \sin \theta - k(x_1-x_2)l_s \]
\[ m \frac{d^2x_2}{dt^2} = -mgx_1 - k(x_1-x_2)l_s \]

\[ 1. \quad \frac{d^2x_1}{dt^2} = -\frac{g}{l} x_1 - \left( \frac{k l_s}{m} \right) (x_1-x_2) \]

\[ 2. \quad \frac{d^2x_2}{dt^2} = -\frac{g}{l} x_2 - \left( \frac{k l_s}{m} \right) (x_2-x_1) \]

To solve - use following procedure:

Add 1 & 2

\[ \frac{d^2(x_1+x_2)}{dt^2} = -\frac{g}{l} (x_1+x_2) + 0 \]

Looks like a system that oscillates as shown.

No spring coupling is present.

\[ \omega = \sqrt{\frac{k}{m}} \]
\[ x_\Sigma = x_1 + x_2 \]
\[ \frac{d^2x_\Sigma}{dt^2} = -\omega^2 x_\Sigma \]

For sum.
Coupled Systems (p.2)

**Subtract 1 & 2**

\[ 4 \quad \frac{d^2 (x_1 - x_2)}{dt^2} = -\frac{g}{m} (x_1 - x_2) - \frac{2k}{m} (x_1 - x_2) \]

\[ = -\left( \frac{g}{m} + \frac{2k}{m} \right) (x_1 - x_2) \]

\[ \omega_D = \sqrt{\frac{g}{m} + \frac{2k}{m}} \]

For difference

**Spring Stretches 2(x_1 - x_2)**

\[ x_\Delta = x_1 - x_2 \]

\[ \frac{d^2 x_\Delta}{dt^2} = -\omega_D^2 x_\Delta \]

**Note**

\[ x_\Sigma = x_1 + x_2 \]

\[ x_\Delta = x_1 - x_2 \]

\[ x_1 = \frac{x_\Sigma + x_\Delta}{2} \]

\[ x_2 = \frac{x_\Sigma - x_\Delta}{2} \]

\[ x_\Sigma = A_\Sigma \cos \omega_\Sigma t + B_\Sigma \sin \omega_\Sigma t \] \[ \text{Solution to (3)} \]

\[ x_\Delta = A_\Delta \cos \omega_\Delta t + B_\Delta \sin \omega_\Delta t \] \[ \text{Solution to (4)} \]

**Initial Condition determines** \( A_\Sigma, B_\Sigma, A_\Delta, B_\Delta \)

Use \( \cos \omega_\Sigma t, \cos \omega_\Delta t \) when no initial speeds

Just initial displacement

\[ L x \quad x_1 = A \] at \( t = 0 \) \$ \[ x_2 = 0 \] no initial \$'s

**The write**

\[ x_1 = A_\Sigma \cos \omega_\Sigma t + A_\Delta \cos \omega_\Delta t \]

\[ x_2 = A_\Sigma \cos \omega_\Sigma t - A_\Delta \cos \omega_\Delta t \]
**Coupled Systems (p.3)**

\[
\begin{align*}
\text{at } t = 0 & \quad x_1 = A = A_\Sigma + A_\Delta \\
& \quad x_2 = 0 = A_\Sigma - A_\Delta \\
& \Rightarrow A = 2A_\Sigma \quad A_\Sigma = A/2 = A_\Delta \\
\end{align*}
\]

\[
\begin{align*}
x_1 &= \frac{A}{2} \left( 2 \cos \frac{\omega_\Sigma - \omega_\Delta t}{2} \right) \\
\quad &= \frac{A}{2} \cos \frac{\omega_\Sigma - \omega_\Delta t}{2} \\
\end{align*}
\]

\[
\begin{align*}
x_2 &= \frac{A}{2} \left( 2 \cos \frac{\omega_\Sigma + \omega_\Delta t}{2} \right) \sin \left( \frac{\omega_\Sigma - \omega_\Delta t}{2} \right) \\
\end{align*}
\]

\[
\omega_\Sigma = \sqrt{\frac{g}{l}} \\
\omega_\Delta = \sqrt{\frac{g + 2k_0 l}{m}}
\]

When \( x_1 \) term \( \cos \frac{\omega_\Sigma - \omega_\Delta t}{2} = \pm 1 \):

\[
\sin \frac{\omega_\Sigma - \omega_\Delta t}{2} = 0, \pi, 2\pi
\]

\( x_1, x_2 \) oscillate out of phase

Time for \( x_0 \) to reach 1st maximum in oscillations:

\[
\sin \frac{\omega_\Sigma - \omega_\Delta t}{2} = \pm 1 \quad \left| \frac{\omega_\Sigma - \omega_\Delta t}{2} \right| = \frac{\pi}{2}
\]

Then \( x_1 \sim \cos \frac{\omega_\Sigma - \omega_\Delta t}{2} = 0 \)