LECTURE 6  PHYSICS 201

These notes already appeared in previous lectures — the topics were not covered.

Topics:

Doppler effect — shock waves

The wave equation for waves on a string —

Parallels of ion equation with planetary motion & motion of an electron around a proton —

Wave length on a string —

Spectrum of hydrogen

Examples of simple harmonic motion
Doppler Effect

What happens to frequency $f$ and wavelength when source is moving, when observer is moving - both moving.

Stationary Case

A stationary source emits a harmonic wave vibrating at frequency $f$ with period $T$. The wave propagates with speed $v$. In one period $T$ the wavefront has advanced $\lambda$:

$$v \cdot T = \lambda$$

$$T = \frac{1}{f}$$

$$v = f \cdot \lambda$$

But $v = \lambda f$.

Perception of $f$ can be altered is source moves or observer moves.

Stationary Case

$\lambda \rightarrow \lambda'$

Source SR towards

$\nu_{SR} = 0$

Source VS away

$\nu_{VS} = 0$

Cases.

Can combine both.
$\lambda = \frac{D}{v}$

Time it take for sound to go from source to observer = $D/v$

$\lambda'$ is reduce, $f'$ is increased

$D' = D - \frac{v_{sr}t}{v} = D - D\frac{v_{sr}}{v} = D(1 - \frac{v_{sr}}{v})$

$\lambda' = D(1 - \frac{v_{sr}}{v}) = 1 - \frac{v_{sr}}{v} \quad \lambda' \downarrow$ shorter

$f' = \frac{v}{v_{sr}} = \frac{v}{v_{sr}} = \frac{f_0}{v_{sr}} \uparrow$

$\lambda f_0 = v$

If source moves away

$\lambda'$ is stretch out

$\lambda' = \lambda_0(1 + \frac{v_{sr}^2}{v^2}) \quad f' = \frac{v}{1 + \frac{v_{sr}^2}{v^2}} = \frac{f_0}{1 + \frac{v_{sr}^2}{v^2}} \downarrow$
CASE 2. OBSERVER MOVES, $S'$ SOURCE STATIONARY.

$U \rightarrow K \rightarrow 1 \rightarrow \frac{U}{25_{ob}}$

$U \rightarrow \frac{U}{ob}$

NOTHING HAPPENS TO $\lambda$.

OBSERVER SEES CREST COMING FASTER THAN WHEN HE/SHE IS NOT MOVING.

CREST 1 ARRIVES AT $t$ IF YOU DON'T MOVE CREST 2 ARRIVES AT $D = \lambda = Ut, t = T$ PERIOD $= \frac{1}{f'}$

$T'$ IS WHEN YOU SEE PEAK (2)

\[ \frac{(U + U_{ob})}{U_{ob}} T' = \lambda' \]

RELATIVE \( \frac{U}{U_{ob}} \)

\[
\begin{align*}
\lambda' &= \frac{U}{(U + U_{ob})} \\
\lambda' &= U(1 + \frac{U_{ob}}{U}) \\
f' &= f(1 + \frac{U_{ob}}{U}) \\
f' &= \frac{U}{25_{ob}} = \lambda' = \frac{\lambda}{25_{ob}} = f_0 (1 + \frac{U_{ob}}{U}) \\
\text{MOVING AWAY} & \quad f' = f_0 (1 - \frac{25_{ob}}{U})
\end{align*}
\]
Summary of Doppler

Find the condition — Multiply each

<table>
<thead>
<tr>
<th>MOVING SOURCE</th>
<th>MOVING OBSERVER</th>
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<tbody>
<tr>
<td>( u_{SR} \rightarrow u \rightarrow v = 0 )</td>
<td>( u \rightarrow u \rightarrow v_{obs} )</td>
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</table>

WAVE \( \lambda \) COMPRESSED

\[
\frac{f_{SR}}{u_{SR}} = \frac{f_0}{1 - \frac{u_{SR}}{u}}
\]

\[
T' = \frac{T_0}{1 + \frac{u_{SR}}{u}}
\]

\( \lambda \) STRETCHED

\[
\frac{f_{SR}}{u_{SR}} = \frac{f_0}{1 + \frac{u_{SR}}{u}}
\]

\[
T' = \frac{T_0}{1 - \frac{u_{obs}}{u}}
\]

Rescale \( \lambda \)  
Same \( u \).

\( \lambda' f' = u \)

Use same \( \lambda \)
Use \( u \) RELATIVE.

\( \lambda' f' = u \) RELATIVE

\( \lambda f' = u \)
Doppler Shift from Moving & Reflecting Wall

Reflecting Wall Acts First as a Moving Observer Which Then Reemits Sound Back to Original Origin as a Moving Source

Step 1 Source \( f_0 \)\( \rightarrow \) \( f' \)\( \leftarrow \) \( f'' \)\( \rightarrow \) \( f_0 \)

Wall Acts as Moving Observer Sees \( f' \)

Because of \( u \) \( \text{rel.} \)

Step 2 Moving Source for Reflected Wave \( f'' \) Related to \( f' \)

Source is Now Moving Observer for Reflected Wave

Step 1 Moving Observer Toward Source

\[ f' = f_0 \left(1 + \frac{u_{\text{obs}}}{c}\right) \]

Step 2 Reflected Wave—Moving Source Toward Stationary Observer

\[ f'' = \frac{f'}{1 - \frac{u_{\text{sr}}}{c}} \]

\[ v_{\text{sr}} = v_{\text{obs}} = v_{\text{object}} \]

\[ f'' = f_0 \cdot \frac{1 + \frac{v_{\text{object}}}{c}}{\left(1 - \frac{v_{\text{object}}}{c}\right)} \approx f_0 \left(1 + \frac{2v_{\text{object}}}{c}\right) \]

If \( v_{\text{object}} < c \)

\[ \frac{f'' - f_0}{f_0} = \frac{2u}{c} = \frac{v}{c} \]

"Beat frequency \( f' - f'' \) missing..."
Doppler Radar sends out a microwave (EM wave) measures $f''$ to $f_0$ of reflected wave to get $\mathbf{\text{object}}$

$\lambda: \text{1 mK} \leq \lambda \leq \text{1 m}$

$\text{RADAR \sim MICROWAVE} \quad 300 \text{GHz} \leq f \leq 36 \text{GHz} = 300 \text{MHz}$

$u_{\text{sound}} \rightarrow c \quad \text{s}peed \text{ of light} \quad u_{obj} = u''$

$f'' = \frac{f(1 + u'/c)}{1 - u'/c}$

Used in weather & police speed guns
Shock Wave:

Stationary Source Emits 4 Waves Period T

Wave Front Emited at L = 4T

Moving Source

New Center for Wave Front Distance Moved U_s T

U_s T MOVING TO RIGHT

This point is inside U_s AT CIRCLE

\[ d_4 = U_s 4T \]

Shock Cone 4 \( \theta_s \)

\[ \sin \theta_s = \frac{U_{\text{SHOCK}}}{U_{\text{SOURCE}}} \]

Shock Wave

This point is outside \( U_{4T \text{SOURCE}} \)
THE WAVE EQUATION - STRING.

TENSION IS ALONG THE STRING.

THE SLOPE OF THE STRING AT ANY POINT IS \( \frac{dy}{dx} \).

Why \( \frac{dy}{dx} \)?

Easy to see why \( \frac{dy}{dx} = 0 \), \( F = ma \).

IF \( T \) AT (A) IS \( \neq \) TO \( T \) AT (B), THE STRING WOULD NOT RETURN TO HORIZONTAL POSITION.

THE STRING HAS TO HAVE CURVATURE.

THE SLOPE IS \( \frac{dy(x,t)}{dx} \).

THE CURVATURE IS \( \frac{d^2y(x,t)}{dx^2} \) \( \rightarrow \) \( \frac{2^2y(x,t)}{2x^2} \).

THE FACTOR \( \frac{5y(x,t)}{dt^2} \) COMES FROM THE ACCELERATION.

\( T_n - \frac{dy}{dx} \bigg|_{x} + \frac{dy}{dx} \bigg|_{x+dx} \)

\( F_y = T \sin \theta \approx T_0 \sin \theta \approx T \tan \theta \).

\( \tan \theta = \frac{dy}{dx} \)

\( F_y = T \frac{dy}{dx} \bigg|_{x} - T \frac{dy}{dx} \bigg|_{x+dx} \)

\( \frac{dy}{dx} \bigg|_{x+dx} - \frac{dy}{dx} \bigg|_{x} = \frac{M}{l} \cdot dx \cdot \frac{\frac{dy}{dt}}{dt} \)

\( \frac{dy}{dx} \bigg|_{x+dx} - \frac{dy}{dx} \bigg|_{x} = \frac{d^2y}{dx^2} \)

\( \frac{d^2y}{dx^2} \approx \frac{d^2y}{dt} \), \( \frac{2^2y}{2x^2} \), \( \frac{2y}{2x} \), \( \frac{T}{l} \).
Another view - Waves on a string, and V

\[ a = \frac{v^2}{R}, \quad \Delta M = \frac{ML}{2} \]

\[ e = R(2\theta) \]

\[ F_y = 2TS \sin \theta \approx 2T \theta = \Delta M a. \]

\[ 2T \theta = \frac{M2GRv^2}{R} \]

\[ v^2 = \frac{T}{M/L} \]
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<tr>
<th>MUSICAL INSTRUMENTS</th>
<th>MUSIC OF THE PLANETS</th>
<th>SONG OF THE ELECTRONS</th>
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<tr>
<td>STRINGS/RINGS</td>
<td>PLANETARY MOTION</td>
<td>BOHR'S HYDROGEN ATOM</td>
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**Parallels**

- **Musical Instruments**
  - **Strings/Rings**
  - **Waves**

- **Music of the Planets**
  - **Planetary Motion**
  - **Circular Orbit**
  - **OMO**

- **Song of the Electrons**
  - **Bohr's Hydrogen Atom**
  - **Proton**
  - **Electron**
  - **Electric Potential Energy**
  - **Total Energy**

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**String**

- **A**
- **B**
- **L**

\[ \lambda_f = V_T \quad T_f = \frac{1}{f} \]

\[ n\lambda = 2L \]

\[ \lambda_n = \frac{2\pi R}{n} \]

\[ f_n = \frac{V_T}{\lambda_n} = n\left(\frac{V_T}{2\pi R}\right) \]

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**Waves on a Circle**

\[ R \]

\[ T = \frac{4\pi^2 R}{V_T^2} \]

\[ T = \frac{4\pi^2 R^3}{GM_0} \]

\[ \lambda = \frac{2\pi R}{n} \]

\[ T_q = \frac{2\pi R}{V_T} \]

\[ T_q = \frac{2\pi R^2}{V_T} \]

\[ T_q = \frac{2\pi R}{V_T} \]

\[ T_q = \frac{2\pi R^2}{V_T} \]

\[ T_q = \frac{2\pi R^3}{V_T} \]

\[ T_q = \frac{2\pi R^4}{V_T} \]

\[ T_q = \frac{2\pi R^5}{V_T} \]

\[ T_q = \frac{2\pi R^6}{V_T} \]

\[ T_q = \frac{2\pi R^7}{V_T} \]

\[ T_q = \frac{2\pi R^8}{V_T} \]

\[ T_q = \frac{2\pi R^9}{V_T} \]

**Bohr's Atom**

\[ \lambda = \frac{\hbar}{p} \]

\[ p = m_v \]

\[ \hbar \]

**DeBroglie Lambda**

\[ \lambda = \frac{\hbar}{p} \]

\[ p = m_v \]

\[ \frac{3}{2} \text{ power} \left( \frac{\hbar}{m_v} \right) \]

\[ \lambda = \frac{2\pi R}{n} \]

**Momentum**

\[ p = m_v \]

\[ \frac{3}{2} \text{ power} \left( \frac{\hbar}{m_v} \right) \]

\[ L = n\lambda \]
Bohr's Quantization of Energy

1. \[ L = \frac{mv^2}{c} R = n^2 \frac{\lambda}{R} \]
   \[ \lambda = \frac{\hbar}{p} \quad p = mv \]

   \[ F = \frac{mv^2}{R^2} = \frac{1}{4\pi\varepsilon_0} \frac{e^2}{R^2} \quad (F = ma) \]

   \[ V = \frac{1}{4\pi\varepsilon_0} \frac{e^2}{mR} \]

   \[ F = mg \]

2. \[ V = \frac{n^2 e^2}{m^2 R^2} \]

   \[ \text{Square (1) and solve for } V^2 \]

   \[ L = n^2 \frac{\lambda}{R} = \frac{n^2}{R} \]

   \[ V^2 = \frac{n^2 \lambda^2}{m^2 R^2} \]

   \[ V = \frac{n^2 \lambda}{mR} \]

   \[ \text{Substitute into (2) and solve for } R \]

   \[ R = \frac{n^2 \lambda^2}{m e^2} = n^2 a_0 \]

   \[ \text{Total } E = -\frac{1}{2} \left| PE \right| = -\frac{1}{2} \frac{e^2}{4\pi\varepsilon_0 R} \]

   \[ = -\frac{1}{2} \frac{e^2}{4\pi\varepsilon_0 a_0} \frac{1}{n^2} \]

   \[ \text{Kepler's Law} \]

   \[ T = \frac{1}{2} = \frac{2\pi}{V} \sqrt{\frac{m e^2}{4\pi\varepsilon_0} \frac{1}{n^2}} \]

   \[ R = n^2 a_0 \]

   \[ T \propto n^3 \quad T \propto \frac{1}{n^2} \]

   \[ T = \frac{1}{n^3} \]
EXAMPLES OF SHM.

T = mg

$\sum F_x$

$T \sin \theta + F = \frac{mg}{d} + kx$

$x = A \cos \omega t$

$\frac{d}{dt}^2 (\frac{mg}{d} + kx) = \frac{mg}{d} \frac{x}{d} + \frac{k}{m} x$

$\theta \approx 0 \cos \theta \approx 1$

$T \cos \theta - mg = 0 \quad T = mg$

$T \sin \theta = m \frac{d^2 x}{dt^2}$

$\sin \theta = \frac{x}{d}$

$s = \frac{x}{d}$

$s = \frac{-x}{d}$

$-mg \frac{x}{d} + mg \frac{x}{d} = \frac{mg}{d} \frac{x}{d} + \frac{k}{m} x$

$k = mg/d = \frac{9}{2}$
EXAMPLES OF SHM.

\[ P = \rho w g h = \frac{E}{\frac{\rho w h A}{h}} A \]

\[ hA = V \]

\[ F = \rho h w A g \text{ weight of water from } x \text{ to top.} \]

\[ F = \rho x A = \frac{\rho x A}{h} = \frac{\rho x A}{h} \]

\[ P_0 = \frac{x_0}{h} \]

\[ P = \frac{x}{h} \]

\[ \rho g x_0 A = Mq \]

\[ \frac{\rho g x_0 A}{h} = P = \frac{\rho x A}{h} \]

\[ (x+x)Agw - xAgw = \frac{m d^2 x}{dt^2} = \frac{Agx}{h} \]

INCLUDE SIGN \[ \frac{Agx}{h} \]

\[ \omega_0 = \sqrt{\frac{\rho g}{\rho c}} = \sqrt{\frac{AgRN}{\frac{\gamma P}{\rho c}}} = \sqrt{\frac{AgRN}{\frac{\gamma P}{\rho c}}} \text{ PA-Mg = Mc \frac{d^2 x}{dt^2}} \]

\[ \omega_0 = \sqrt{\frac{\gamma PA^2}{\rho = \frac{PA}{\rho} \frac{d^2 x}{dt^2}}} \text{ RESONING FORcing} \]
SHEM & RESONANCES.
PARALLEL WITH ELECTRICAL CIRCUITS.

\[ \omega_0 = \sqrt{\frac{k}{m}} \]
\[ \omega_0 = \frac{1}{\sqrt{LC}} \]

MECHANICAL

\[ \ddot{x} = \frac{d^2x}{dt^2} \]
\[ a = \frac{d^2x}{dt^2} = \frac{dv}{dt} \]
\[ f = -\gamma mv \]
\[ m \ddot{x} + \gamma m \frac{dx}{dt} + kx = F_{\text{ext}} \]

ELECTRICAL

\[ i = \frac{dQ}{dt} \text{ CURRENT} \]
\[ L \frac{di}{dt} + IR + \frac{Q}{C} = V_{\text{ext}} \]

ROTATIONAL

\[ \dot{\theta} = \frac{d\theta}{dt} \]
\[ \ddot{\theta} = -\kappa \dot{\theta} \]

\[ \int \frac{1}{L} \]