THE WAVE EQUATION - STRING.

TENSION IS ALONG THE STRING.
THE SLOPE OF THE STRING
AT ANY POINT IS \( \frac{dy}{dx} \).

IF \( T \) AT A IS AT TO T AT B THE STRING WOULD
NOT RETURN TO HORIZONTAL POSITION
THE STRING HAS TO HAVE CURVATURE

THE SLOPE IS \( \frac{dy(x,t)}{dx} \)

THE CURVATURE IS \( \frac{d^2 y(x,t)}{dx^2} = \frac{\partial^2 y(x,t)}{\partial x^2} \)

THE FACTOR \( \frac{\partial^2 y(x,t)}{\partial t^2} \) COMES FROM THE ACCELERATION

\[ T \left[ \frac{dy}{dx} \right]_{x} + \frac{dy}{dx} = \frac{\partial y}{\partial x} + \frac{dy}{dx} \left[ \frac{dy}{dx} \right]_{x} \]

\[ F_y = T \sin \theta \approx Ts \approx T \tan \theta, \quad \tan \theta = \frac{dy}{dx} \]

\[ F_y = T \left( \frac{dy}{dx} \right)_{x} - T \left( \frac{dy}{dx} \right)_{x} = \frac{M dx \cdot dx}{\mu \ dx} = \frac{\partial y}{\partial t} \left( \frac{2 \ y}{dx^2} \right) \]

\[ \frac{dy}{dx} = \frac{dy}{dx} \frac{dx}{dx} = \frac{\partial^2 y}{\partial x^2} \]

\[ T \left( \frac{dy}{dx^2} \right) = \frac{M \ 2^y}{\partial t^2} \quad \frac{\partial^2 y}{\partial x^2} = \frac{1}{2} \left( \frac{\partial y}{\partial x} \right)^2 \quad \delta = \frac{T L}{M} \]
ANOTHER VIEW - WAVES ON A STRING AND V

\[ a = \frac{v^2}{R}, \quad \Delta M = \frac{M}{L} \]

\[ L = R(2\theta) \]

\[ F_y = 2T \sin \theta = 2T \theta = \Delta M a. \]

\[ 2T \theta = \frac{M}{l} 2 \theta R \frac{v^2}{R} \]

\[ v^2 = \frac{T}{\mu/L} \]
<table>
<thead>
<tr>
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<th>MUSIC OF THE PLANETS</th>
<th>SONG OF THE ELECTRONS</th>
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<td>STRINGS/RINGS</td>
<td>PLANETARY MOTION</td>
<td>BOHR'S HYDROGEN ATOM</td>
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<tr>
<td>2TS\cdot\theta = m\alpha</td>
<td>F = G\frac{M}{R^2} = m\frac{V^2}{R}</td>
<td>F_e = \frac{1}{2} \frac{e^2}{4\pi\varepsilon_0 R}</td>
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<td>\alpha = \frac{V^2}{R}</td>
<td>\frac{V_p}{\frac{G M}{R^2}} = \frac{V}{R}</td>
<td>\frac{V_e}{(\frac{1}{4\pi\varepsilon_0 \frac{e^2}{m c}})} = \frac{1}{R_c}</td>
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<tr>
<td>m = \frac{M}{L} \quad \ell = R\theta</td>
<td>V^2 = \frac{G M}{R}</td>
<td>\sqrt{\frac{1}{2} \frac{e^2}{4\pi\varepsilon_0 R}}</td>
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<td>\frac{V_T^2}{T} = \frac{L}{M}</td>
<td>V_p = \frac{GM}{R}</td>
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<td>STRING</td>
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<td>\lambda_f = V_T \quad \frac{T_f}{2} = 1</td>
<td>T = \frac{2\pi R}{V}</td>
<td>T_Q \sim \frac{\lambda}{2}</td>
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<td>\nu \lambda = 2\lambda ; \nu = \frac{V_T}{2L}</td>
<td>T_Q = \frac{2\pi R}{V_p}</td>
<td>\lambda = \frac{\hbar}{P_e} ; P_e = \frac{m e}{c}</td>
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<td>WAVELENGTH</td>
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<tr>
<td>\lambda_n = 2\pi R/n</td>
<td>T_Q = \frac{2\pi R}{V_p}</td>
<td>n \lambda = \frac{2\pi R}{n}</td>
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Bohr's Quantization of Energy

1. \[ L = m_e v_e R = n^2 \frac{\hbar}{2} \]

2. \[ F = \frac{m_e v_e^2}{R} = \frac{1}{4 \pi \epsilon_0} \frac{e^2}{R} \quad (F = ma) \]

3. \[ V_e^2 = \frac{1}{4 \pi \epsilon_0} \frac{e^2}{m_e R} \]

Square (1) and solve for \( V_e^2 \)

4. \[ \frac{V_e^2}{\hbar^2} = \frac{n^2 a_0^2}{m_e^2 R^2} \]

Substitute into (2) and solve for \( R \)

5. \[ \frac{n^2 a_0^2}{m_e^2 R^2} = \frac{e^2}{4 \pi \epsilon_0} \frac{1}{m_e R} \]

6. \[ R = \frac{n^2 a_0}{\sqrt{\frac{e^2}{4 \pi \epsilon_0} m_e}} = n^2 a_0 \]

Total \( E = -\frac{1}{2} |p|^2 = -\frac{1}{2} \frac{e^2}{4 \pi \epsilon_0 R} \)

7. \[ E = -\frac{1}{2} \frac{e^2}{4 \pi \epsilon_0 a_0} \frac{1}{n^2} \]

Kepler's Law

8. \[ T_0 = \frac{1}{f_0} = \frac{2 \pi R}{v_e} = \frac{3}{2} \frac{R}{\sqrt{\frac{e^2}{4 \pi \epsilon_0} m_e}} = n^2 a_0 \]

9. \[ R = n^2 a_0, \quad R^{3/2} = n^3 a_0^{3/2} \]

10. \[ T_0 \sim n^3, \quad f_0 \sim \frac{1}{n^3} \]
BEATS.

RECALL - STANDING WAVE RESULTED IN ADDING 2 SINE WAVE ONE $\Rightarrow$ & ONE REFLECTED $\leftarrow$

USED $\sin A + \sin B = 2 \sin \left( \frac{A + B}{2} \right) \cos \left( \frac{A - B}{2} \right)$, A TRIG IDENTITY

$\cos \left( \frac{A - B}{2} \right) = \cos \left( \frac{B - A}{2} \right)$

IN BEATS YOU HAVE TWO FREQUENCIES PRESENT AT THE SAME TIME.

ASIN $\omega t$ HAS NOT $\omega t$. TAKE EQUAL AMPLITUDES TO BEGIN WITH.

NODE

ALSO

$A \cos \omega t + A \cos \omega t = 2A \cos \left( \omega t + \frac{\omega t}{2} \right) \cos \left( \frac{\omega - \omega t}{2} \right)$

TO BE DISCUSSED LATER.

SIMILAR RESULTS APPEAR IN COUPLED PENDULUM

$\omega = \sqrt{\frac{k}{m}}$  $\omega = \sqrt{\frac{g}{l} + \frac{2k}{m}}$

OSCIILATES. AT REST TO BEGIN WITH THEN SLOWS DOWN THEN STOPS. THEY START TO STOP STOPS.
BEATS.

Adding harmonic waves of different $\omega$'s, which are close to each other.

$$A \cos \omega_1 t + A \cos \omega_2 t = A(\cos \omega_1 t + \cos \omega_2 t)$$

$$= 2A \cos \frac{\omega_1 + \omega_2}{2} t \cos \frac{\omega_2 - \omega_1}{2} t$$

$\omega + \omega_2 - \omega \frac{\omega_2 - \omega_1}{2}$ determines $\omega_0$.

When $\frac{\omega_2 - \omega_1}{2}$ is $\pi$, $3\pi$, $5\pi$, etc.,

$$\text{COS} \frac{\omega_2 - \omega_1}{2} t = 0$$

Determines beats.

$$\frac{\omega_2 - \omega_1}{2} = \pi$$

Period for beats.

$$\frac{2\pi}{f_B} = \frac{2\pi}{B} = \omega = \omega$$

$$\omega = \left| \frac{\omega_2 - \omega_1}{2} \right| f_{\text{beat}} = \frac{1}{f_2 - f_1}$$

The factor $\frac{1}{2}$ goes away because you hear the envelope modulation for $\cos \left( \frac{\omega_2 - \omega_1}{2} t \right)$ intensity variations.
\[ R = A \cos(k_1 x - \omega_1 t) + A \cos(k_2 x - \omega_2 t) \quad \frac{\omega}{k_1} = \frac{\omega}{k_2} \]

\[ = 2A \cos \left( \frac{k_1 + k_2}{2} x - \frac{\omega_1 + \omega_2}{2} t \right) \cos \left( \frac{k_1 - k_2}{2} x - \frac{\omega_1 - \omega_2}{2} t \right) \]

\[ \omega_1 = \frac{\omega_1 + \omega_2}{2} \quad \frac{k_1}{2} = \frac{k_1 + k_2}{2} \quad \frac{\omega_2}{k_2} = \frac{\omega_1 + \omega_2}{2} \]

\[ \omega_2 = \frac{\omega_1 - \omega_2}{2} \quad \frac{k_2}{2} = \frac{k_1 - k_2}{2} \quad \frac{\omega_1 - \omega_2}{k_2} = \frac{\Delta \omega}{\Delta k} \]

\[ \frac{\omega}{k} = \text{Group Velocity} = \frac{d(\omega)}{dk} \]

\[ \text{In this case} \quad \frac{\omega}{k} = \frac{\omega_1}{k_2} \quad \frac{\omega_1}{k_1} \quad \frac{\omega_2}{k_2} \]

\[ \frac{\omega_1 - \omega_2}{k_1 + k_2} = \frac{\omega_1 - \omega_2}{k_1 + k_2} = \frac{\Delta \omega}{\Delta k} \quad \text{ Called Phase Velocity} \]

\[ \frac{\omega_1 - \omega_2}{k_1 + k_2} - \frac{\omega_1}{k_2} = \frac{\omega_1 - \omega_2}{k_2} = \frac{\omega_1 - \omega_2}{k_1 - k_2} \]

\[ \text{For Phase} \]
**BEATS - UNEQUAL FREQUENCIES & AMPLITUDES**

**Exponential Representation**

\[ e^{i(\omega_1 + \omega_2) t} \]

\[ e^{i\omega_1 t} \rightarrow A_1 e^{i\omega_1 t} \]

\[ e^{i\omega_2 t} \rightarrow A_2 e^{i\omega_2 t} \]

\[ (\omega_2 - \omega_1) t \]

\[ \omega_1 t \rightarrow A_1 e^{i\omega_1 t} \]

\[ \omega_2 t \rightarrow A_2 e^{i\omega_2 t} \]

**Use Law of Cosines to Find Resultant**

\[ A_k^2 = A_1^2 + A_2^2 - 2A_1 A_2 \cos(180 - (\omega_2 - \omega_1) t) \]

\[ = A_1^2 + A_2^2 + 2A_1 A_2 \cos(\omega_2 - \omega_1) t \]

\[ A_k \text{ oscillates from } A_1 + A_2 \text{ when } \cos(\omega_2 - \omega_1) t = +1 \]

\[ A_k^2 = A_1^2 + A_2^2 + 2A_1 A_2 = (A_1 + A_2)^2 \text{ vectors } A_1 + A_2 \]

\[ \text{to } |A_2 - A_1| \text{ when } \cos(\omega_2 - \omega_1) t = -1 \]

\[ A_k^2 = A_1^2 + A_2^2 - 2A_1 A_2 = (A_2 - A_1)^2 \text{ vectors } A_2 - A_1 \]

\[ \text{cos}(\omega_2 - \omega_1) t = -1 \text{ at } (\omega_2 - \omega_1) t = \pi, 3\pi, ... \]

**Show Figures From Mathematica**
\( f[A1_, A2_, \omega1_, \omega2_, t_] := A1 \cdot \cos[\omega1 \cdot t] + A2 \cdot \cos[\omega2 \cdot t] \)

\( \text{Plot}[f[1, 1, 50, 45, t], \{t, 0, 5\}] \)

\( \left| \omega_1 - \omega_2 \right| t = \frac{\pi}{2}, \frac{3\pi}{2} \)

\( \frac{5}{2} t = \frac{\pi}{2} \)

\( t = \frac{\pi}{5} \approx 0.63 \)

\( \frac{5}{2} t = \frac{3\pi}{2} = 3 \times 0.63 \approx 1.9 \)

\( \frac{\omega_1 - \omega_2}{2} T_B = \pi \)

\( T_B = \frac{2\pi}{\omega_1 - \omega_2} = \frac{2\pi}{3\pi/2} = 1.26 \)

\( f_B = \frac{1}{T_B} = \frac{\omega_1 - \omega_2}{2\pi} \)

\( 2\pi f_B = (\omega_2 - \omega_1) \)

\( f_B = f_2 - f_1 \)
PHYSICS OF SOUND - BEATS - Plots Using Mathematica

In[2]:= f[A1_, A2_, w1_, w2_, t_] := A1 \* Cos[w1 \* t] + A2 \* Cos[w2 \* t]

In[3]:= Plot[f[1, 1, 50, 45, t], {t, 0, 5}]

Out[3]=

In[7]:= Plot[f[2, 1, 50, 45, t], {t, 0, 5}]

Out[7]=

In[9]:= Plot[f[1, 1, 50, 25, t], {t, 0, 5}]

Out[9]=
Doppler Effect

What happens to frequency & wavelength when source is moving, when observer is moving - both moving.

Stationary case

A stationary source emits a harmonic wave vibrating at frequency $f$ with period $T$. The wave propagates with speed $v$. In one period $T$, the wavefront has advanced $\lambda$:

$$v \cdot T = D \quad T = \frac{1}{f} \quad \frac{D}{f} = D = \frac{v}{f}$$

But $v = \lambda f$.

Perception of $f$ can be altered if source moves or observer moves.

Stationary case

\[ \lambda' = \lambda \]

Cases.

Source $\nabla$ toward

Source $\nabla$ away

$V_{\text{source}} = 0$

$V_{\text{observer}} = 0$

Can combine both.
$\lambda' = \frac{D}{D' - \frac{u_{sr}}{c}} = \frac{D}{D - \frac{u_{sr}}{c}} = D \left(1 - \frac{u_{sr}}{c}\right)$

$\lambda' = \lambda \frac{D}{D' - \frac{u_{sr}}{c}} = 1 - \frac{u_{sr}}{c}$

$\lambda' f' = \frac{u}{c} = \frac{u}{c} = \frac{f_0}{\lambda_0}$

$\lambda_0 f_0 = \frac{u}{c} = \frac{f_0}{\lambda_0}$

If source moves away

$\lambda' = \lambda \left(1 + \frac{u_{sr}}{c}\right)$

$f' = \frac{u}{c} = \frac{f_0}{\lambda_0}$

$t = \text{time it take for sound to go from source to observer} = \frac{D}{u}$

$\lambda$ is reduced $\lambda'$ is increased

$\frac{u_{sr}}{c}$
Case 2: Observer moves, $s'$, source stationary.

$$\lambda'$$

Nothing happens to $\lambda$.
Observer sees crest coming faster than when he/she is not moving.

Crest 1 arrives at $t$ if you don't move. Crest 2 arrives at $D = 2 = \frac{u' t}{t}$.

$t = T$, period $= \frac{\lambda}{u'}$.

$T'$ is when you see peak 2.

$$\left(\frac{u + u_{ob}}{u_{ob}}\right)T' = \lambda$$

Relative $u_{ob}$.

$$T' = \frac{f'}{f} = \frac{\lambda}{u' + u_{ob}}$$

$$f' = \frac{u + u_{ob}}{2} = \frac{\lambda (1 + \frac{u_{ob}}{u'})}{\lambda} = f_0 \left(1 + \frac{u_{ob}}{u'}\right)$$

Moving away, $f' = f_0 \left(1 - \frac{u_{ob}}{u'}\right)$. 
**Summary of Doppler**

**Find the condition - Multiply each**

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<thead>
<tr>
<th>MOVING SOURCE</th>
<th>MOVING OBSERVER</th>
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<tbody>
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<td>$v_{sR}$</td>
<td>$v_{obs}$</td>
</tr>
<tr>
<td>$\rightarrow$</td>
<td>( \rightarrow )</td>
</tr>
<tr>
<td>$\lambda$ COMPRESSED</td>
<td>$\lambda$ STRETCHED</td>
</tr>
<tr>
<td>$f'<em>{sR} = f_0 \left( 1 - \frac{v</em>{sR}}{v} \right)$</td>
<td>$f'<em>{obs} = f_0 \left( 1 + \frac{v</em>{obs}}{v} \right)$</td>
</tr>
<tr>
<td>$T' = \frac{T}{1 + \frac{v_{obs}}{v}}$</td>
<td>$T' = \frac{T_0}{1 - \frac{v_{obs}}{v}}$</td>
</tr>
</tbody>
</table>
Doppler Shift from Moving & Reflecting Wall

Reflecting wall acts first as a moving observer which then reemits sound back to original origin as a moving source. Wall acts as moving observer, sees $f'$.

Step 1: Source

Moving source for reflected wave with $f''$ related to $f'$.

Step 2: Moving observer toward source

$f' = f_0 \left(1 + \frac{V_{obs}}{V}\right)$.

Step 2: Reflected wave - moving source toward stationary observer

$f'' = \frac{f'}{1 - \frac{V_{obs}}{V}}$.

$V_{sr} = V_{obs} = V_{effective}$

$f'' = f_0 \left(1 + \frac{V_{obs} \Delta f}{V}\right) \approx f_0 \left(1 + \frac{2V_{object}}{V}\right)$

If $V_{object} < V$, \( (f - f_0)^2 = V_{Object} \)

"Beat frequency?"
Doppler radar sends out a microwave (EM wave) measures $f''$ to $f_0$ of reflected wave to get $v_{\text{object}}$

$\lambda: 1\text{ mm} \leq \lambda \leq 1\text{ m}$

Radar $\sim$ Microwave

$300\text{ GHz} \leq f \leq 30\text{ GHz} = 300 \text{ kHz}$

$v_{\text{sound}} \rightarrow C$ speed of light $v_{\text{obj}} \neq 0$

$f'' = \frac{f}{1 + \frac{v}{c}}$

Used in weather, police speed guns
Shock Wave

Stationary Source emits 4 Waves. Period T

Wave front emitted at T = 4T

Moving Source

Wave front emitted at 4T

New center for wave front distance moved

\( u \cdot 4T \)

\( u_{SR} \) moving to right

This point is inside \( u \cdot 4T \) circle

\( d_{4T} = u \cdot 4T \)

Shock cone \( \theta_s \)

\( \sin \theta_s = \frac{u_{source}}{u_{sound}} \)

\( u_{SR} > u_{sound} \)

Shock wave

This point is outside \( u \cdot 4T \) circle

\( \frac{u_{4T}}{u_{source}} \)
Bring - Additional information for lab

Finish - Dimensional analysis - water waves

Start waves, standing waves, wave eq

Read - Chapter 3