LECTURE 25

Topics

Finish: Speech & Singing

1. Go over video from last lecture
2. Discuss pulses from larynx
   a) Uncertainty relation
      \[ \Delta x \Delta p \geq \frac{\hbar}{2} \]
   b) Dirac Delta function limit
3. Formants and the resonances of tubes
   a) Parallel with organ pipes
   b) Parallel with violin
4. Singers format & range of singing

Start: Architectural Acoustics

1. Resonant frequencies of a room
2. Discuss its importance in
   a) Electrons in a metal
   b) Bose-Einstein condensation
   c) Protons & neutrons in a nucleus
   d) Neutron stars
   e) Theory of white dwarf stars
   f) Black body radiation
   g) E&M waves in cavities
   h) Lasers
   i) Many more
   A-G+... All use counting of states
3. Revitalization time & the design of concert halls
   (probably next period)
Partial IV Chapters 23-25

Architectural Acoustics

Resonant Frequencies of a Room - Sides $L_x, L_y, L_z$

Each dimension can be treated the same way that was done for a string.

$$n \lambda_x = 2L_x \quad m \lambda_y = 2L_y \quad p \lambda_z = 2L_z$$

$n = 1, 2, 3; \quad m = 1, 2, 3; \quad p = 1, 2, 3.$

Note if you allow $p = 0$ then you have only waves in the $x, z$ direction - two dimensions; similarly for $n = 0$ or $m = 0$. If you allow $m = 0, p = 0$ then you have waves only in $x$-direction - or 1 dimension.

$$k_x = \frac{2\pi}{\lambda_x} = \frac{2\pi}{2L_x} = n \frac{\pi}{L_x}$$

$$k_y = m \frac{\pi}{L_y} \quad k_z = p \frac{\pi}{L_z}$$
Counting how many states is important in all areas of physics. These include electrons in metals, Bose-Einstein condensation, protons & neutrons in a nucleus, theory of white dwarf stars, black body radiation, electromagnetic waves in cavities, ... to mention a few.

**Procedure - Draw 3-D System - k-space**

\[ \begin{align*}
&\text{The mode } \\
&n = 2, m = 3, \rho = 2 \\
&\text{is a point } \end{align*} \]

\[ n = 2 \]
\[ \lambda_x = \frac{2L_y}{3} \]
\[ \lambda_y = \frac{2L_x}{3} \]

This can be done for every possible mode, then you will get a bunch of dots, filling up the space - below is a two dimensional system.

Note you have 1 dot in an area.

\[ \begin{align*}
&\text{in } k_x \\
&\text{in } k_y \\
&\frac{\pi}{L_x}, \frac{\pi}{L_y} \end{align*} \]
In 3-dimensions - Have one

In a volume, each dot is a distance

\[ \sqrt{\frac{x^2}{l_x^2} + \frac{y^2}{l_y^2} + \frac{z^2}{l_z^2}} \]

A away from the

Next dot in the x - \[ \frac{l_x}{2} \] direction, etc.

\[ \omega_{\text{mp}} = \frac{\sqrt{\frac{x^2}{l_x^2} + \frac{y^2}{l_y^2} + \frac{z^2}{l_z^2}}}{\text{sound}} = \sqrt{\frac{\mathbf{k}}{n_{\text{mp}}}} \]

This equation gives the frequency

For each dot which is associated

With the magnitude of the vector \( \mathbf{k}_{\text{mp}} \).

To simplify matters let \( l_x = l_y = l_z = \frac{L}{2} \) - the box is a cube.

Two questions that come up

1. How many dots (allowed k's, \( \mathbf{k} \)'s, or \( \omega \)'s or \( f = \omega / 2 \pi \)) do you have

   Within a range of \( \mathbf{k} \) from 0 to \( k_{\text{max}} \)

2. How many dots within \( k \) to \( k + dk \)

   See pictures (how many \( \mathbf{\lambda} \) from 1 to 1 + all)
The number of dots are within \( \frac{1}{6} \) of a sphere \((l_x, l_y, l_z)\) all must be greater than 0.

Thus \( \frac{1}{8} \frac{4\pi}{3} r^3 \) is the volume.

Each dot occupies an intrinsic volume
\[
\left( \frac{\pi}{2x} \right) \left( \frac{\pi}{2y} \right) \left( \frac{\pi}{2z} \right) \rightarrow \frac{\pi^3}{8} \frac{r^3}{V_{\text{room}}}
\]

Thus number of dots (states or modes associated with room of volume \( V \)) is
\[
\frac{1}{8} \frac{4\pi}{3} r^3 \frac{\pi^3}{V_{\text{room}}} = \frac{1}{6\pi^2} r^3 V_{\text{room}}
\]

Thus
\[
N(\Omega) = \frac{1}{6\pi^2} r^3 V = \frac{1}{6\pi^2} \left( \frac{2\pi r}{\sqrt{3}} \right)^3 V = \frac{4\pi}{3} \left( \frac{r}{\sqrt{3}} \right)^3 V
\]

\[
\frac{r}{\sqrt{3}} = \frac{2\pi}{\lambda} = \frac{2\pi f}{V}
\]

\[
N(f) = \frac{4\pi}{3} \left( \frac{f}{\sqrt{3}} \right)^3 V + \frac{\pi}{4} \frac{s^2 f}{2 \sqrt{3}} + \frac{L'}{8}
\]

\[
adN(f) = \text{number of states or modes per } \text{unit frequency}
\]

\[
= 4\pi f^2 + V + \frac{\pi}{2} \frac{s^2 f}{\sqrt{3}} + \frac{L'}{8}
\]

For \( N(\lambda) \) \( f = \frac{\lambda}{2\pi} \) \( \frac{f}{\sqrt{3}} = \frac{1}{2} \)

\[
N(\lambda) = \frac{4\pi}{3} \frac{1}{2} V + \frac{\pi}{4} \frac{s^2}{\sqrt{3}} + \frac{L'}{8} \lambda \frac{dN(\lambda)}{d\lambda}
\]
ECHOES & REVERBERATION

\[ v_s = 344 \text{ m/s} = 1130 \text{ ft/s} \]

DIRECT SOUND

A source 113 ft away arrives in 0.13 s = 100 ms
Source 11.3 ft away " " 0.01 s = 10 ms
EARLY SOUND - Shortly after echoes arrive

REVERBERATION SOUND: Shortly after early echoes come very close together - 1000's can arrive with spacings as short as 0.01 ms to 1 ms.

[Diagram of sound decay over time]

CONTINUOUS SOURCE OVER INTERVAL \( t \)

IF SOURCE IS CONTINUOUS, REVERBERATION SOUND builds up to an equilibrium level where sound energy being supplied = sound energy lost to absorption

[Diagram of sound decay over time]

REVERBERATE: DECAYS AWAY TO ABSORBING PROCESSES IN WALLS, CEILING, OBJECTS THAT FILL THE ROOM. OPEN WINDOWS & DOORS.

DECAY Follows an exponential decay law

\[ e^{-\lambda t} \]
ECHOES & REVERBERATION

EXPONENTIAL DECAY SEE FIG. 33.4 P. 527

\[ p = P_{\text{pressure}} = A e^{-\lambda t} \]
\[ \lambda = \text{decay constant} \]
\[ e^{2.3025} = 10 \quad \therefore \lambda = 10^{\frac{1}{2.3025}} = 10^{0.4343} \]
\[ p \approx A \cdot 10^{-0.4343t} \]
\[ \log_{10} p \approx \log_{10} A - 0.4343t \]
\[ \text{dB} = 20 \log_{10} p \approx 20 \log_{10} A - 8.686 \lambda t \]

Straight line with negative slope determined by decay constant. Absorption: increasing objects that absorb increases the decay rate.
ECHOES & REVERBERATION

AFTER SOUND SOURCE STOPS REFLECTIONS
877) CONTINUE BUT ABSORPTION LEADS TO
A SITUATION NO LONGER IN EQUILIBRIUM.
1. IF REVERBERANT SOUND DECAYS TO SLOWLY
THE CLARITY OF THE NEXT NOTE IS LOST.
2. IN SPEECH YOU WANT A SHORTER REVERB
TIME THAN IN MUSIC. MUSIC LIKES A
LONGER REVERB TIME TO GIVE A "LIVELY"
FEELING.
3. SOME EXAMPLES: BOSTON SYMPHONY HALL 1.8S;
N.Y. CARNEGIE HALL 1.7S.
FOR CLASSROOM ~ 1.8 IS GOOD.
4. A FORMULA USED TO CALCULATE THE
REVERBERATION TIME IS DUE TO SABIN CALLED

\[ RT_{60} \sim \frac{V}{K} \]

WHERE [VOLUME OF ROOM K V MEASURES AMOUNT OF]
[ENERGY IN ROOM, AREA ARE THE ABSORPTION AREA]
THE \( T \), REFERS TO CHOOSING A REDUCTION
OF 60dB IN THE SOUND PRESSURE
LEVEL, THE CHOICE OF 60dB

100dB TYPICAL LEVEL IN AN
ORCHESTRA

60dB

40dB TYPICAL ROOM BACKGROUND

5. PROBLEM WITH LOW FREQUENCIES
CALLED BASS LOSS PROBLEM
THE LOW FREQUENCIES NEED A MUCH HIGHER
THRESHOLD TO BE HEARD.
SEE FIG. 6.4 P107
THE THRESHOLD FOR HEARING RISES SIGNIFICANTLY
AT LOW F

70dB

20dB

20 200 Hz 2k
Origin of Factor $1/4$ for Arrow Shown

\[ \frac{1}{2} \text{ Away from Opening} \]
\[ \frac{1}{2} \text{ Toward Opening} \]

Think of particles inside a box trying to escape. Particles move in all directions.

Only those with $\mathbf{V} \perp \mathbf{n}$ to surface can pass through opening.

\[ \mathbf{V} \cdot \mathbf{n} = V \cos \theta \]

Solid angle $\sin \theta \, d\theta \, d\phi$.

\[ \frac{1}{2} \int_0^\pi \cos \theta \sin \theta \, d\theta \int_0^{2\pi} d\phi \]

From $-\frac{\pi}{2}$ to $\frac{\pi}{2}$.

\[ \int_0^{\pi/2} \sin \theta \, d\theta \int_0^{2\pi} d\phi \]

Also $\int_0^{\pi/2}$ goes only from $0$ to $\pi/2$, upper hemisphere since particles inside box are trying to get out.

\[ \Phi_{\text{flux}} = \frac{1}{4} \rho \langle V \rangle \]

\[ \rho \text{ density} \quad <V> \text{ mean speed} \]

# of particles crossing unit area per unit time.
ECHOES & REVERBERATION

HOW $RT_0$ ARISES.

LET $E =$ SOUND ENERGY DENSITY, Joules/m$^3$

TOTAL SOUND ENERGY IS $EV$ VOLUME OF ROOM

ENERGY IS BEING LOST AT A RATE $dEV/dt$

LOSS IS GIVEN BY HOW MUCH ENERGY FALLS ON ABSORBING SURFACES WITH EFFECTIVE AREA $A$. INTENSITY $I$ IS AMOUNT OF ENERGY INCIDENT ON A SURFACE AREA (UNIT AREA) PER UNIT TIME. LOSS IS THE

$-AI$

$dEV = -AI$

IF ALL THE SOUND WERE MOVING IN ONE DIRECTION $I$ AND $E$ RELATED

$$E^2 / 2V = I$$

AMOUNT OF ENERGY IN TUBE IS $A = \pi r^2$

$$\frac{EV \pi r^2}{2V} = IAT$$

$E^2 / 2V = T$

HOWEVER, ECHOES COME IN ALL DIRECTIONS

THIS GIVES RISE TO $\frac{1}{2}EUV = I$ \& $AI = \frac{1}{2}EUV A$

$\frac{1}{2}EV = \frac{AU}{2} A$ SEE NEXT PAGE FOR $\frac{1}{2}$ FACTOR

$\frac{dE}{dt} = -\frac{AU}{2V} A$ \& $dE = -\frac{AU}{2V} t + C$

$E = E_0 e^{-(AU/2V)t}$, $\frac{E}{E_0} = 1 - \frac{AU}{2V}$

$\frac{E}{E_0} = e^{-0.4848 \cdot 10^{-6}} = 10^{-6}$
ECHOS & REVERBERATION

\[ R_{60} = 0.161 \frac{V}{A} \]  
\[ V = \frac{6 \times 4 \frac{V}{A}}{4343 \times 344} \]

\[ \frac{3.44}{m \cdot s \cdot N \cdot A} \]

HAVE TO USE METERS. SINCE \( V = 344 \) WAS USED.

How is \( A \) calculated? How to find \( R_{60} \)?

(A) Simple example-
Walls that don't absorb, only reflect, and open windows that don't reflect.

\[ A = \text{area of walls} + \sum \text{area of windows} \]

\[ A = \text{absorption coefficient for walls} \]

(B) Real rooms

Need absorption coefficients \( \alpha \) for each surface. Assume surface \( A_i \) has coefficient \( \alpha_i \) and area \( A_i \). Then

\[ A = \sum \alpha_i \]

Cover every object in room, including walls...

The coefficients can depend on frequency.

(C) Coefficients \( \alpha_i \) are given in Table 23.1, p. 531

Note the smaller the absorption coefficient, the higher the reflection is and thus the longer the reverberation time.

\[ R_{60} = 0.161 \frac{V}{A} \]

Reduce \( A \) as \( R_{60} \) increases.
ECHOES & REVERBERATION TIME

Q, HAS UNITS IN MKS - METRIC SABINE -
BUT IT JUST A NUMBER WHICH HAS TO BE
MULTIPLIED BY THE AREA,

ABSORPTION BY AIR ITSELF, WHICH DEPENDS
ON TEMPERATURE & RELATIVE HUMIDITY.
THE AIR ABSORPTION DEPENDS ON THE
VOLUME OF THE ROOM SINCE THE AIR OCCUPIES
THE VOLUME,

\[ A = A_{\text{surfaces}} + m \cdot V \]

\[ m = 0.012 \]

at 20°C & 80% RELATIVE HUMIDITY.

PEOPLE IN THE AUDIENCE CONTRIBUTE
0.2 TO 0.6 m² DEPENDING ON FREQUENCY.

EXAMPLE: LECTURE HALL THAT IS 20m x 15m x 8m
WALLS - PAINTED CONCRETE BLOCK, CEILING - PLASTER
FLOOR - CARPET. THEN ADD 200 UPHOLSTERED SEATS
OF WHICH 1/2 ARE OCCUPIED USE f = 500 Hz.

WITH CHAIRS & PEOPLE

WALLS: 0.06
CEILING: 0.06
FLOOR: 0.14

WALLS TOTAL AREA.

PERIMETER = 2X15 + 2X20

\[ \sim 34 \]

\[ = 70 - 70 \times 8 = 560 \text{ m}^2 \]

CEILING: \[ 15 \times 20 = 300 \text{ m}^2 \]

FLOOR: \[ 300 \times 0.14 = 42 \text{ m}^2 \]

100 UNOCCUPIED = 100 x 0.39

100 OCCUPIED = 100 x 0.56

WALLS

\[ \alpha = 0.06 \]

\[ V = 2400 \text{ m}^3 \]

\[ n = 0.012 \]

Air

\[ V = 2400 \text{ m}^3 \]

\[ n = 0.012 \]

94 + 39 + 56 + 29 = 218

42

42

\[ V = 2400 \text{ m}^3 \]

\[ \alpha = 0.161 \times 2400 \sim \frac{400}{218} \]

\[ = \frac{34 + 18 + 42}{418} = 0.8 \text{ LARGE} \]

\[ \frac{400}{218} \]

\[ \sim 0.185 \]
TABLE 23.3  Acoustical characteristics of concert halls

<table>
<thead>
<tr>
<th></th>
<th>Year built</th>
<th>Volume (m³)</th>
<th>Floor area (m²)</th>
<th>Number of seats</th>
<th>t₁ (ms)</th>
<th>Floor</th>
<th>Balc.</th>
<th>125</th>
<th>500</th>
<th>2000 Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>Symphony Hall, Boston</td>
<td>1900</td>
<td>18,740</td>
<td>1550</td>
<td>2630</td>
<td>15</td>
<td>7</td>
<td></td>
<td>.2</td>
<td>1.8</td>
<td>1.7</td>
</tr>
<tr>
<td>Orchestra Hall, Chicago</td>
<td>1905</td>
<td>15,170</td>
<td>1855</td>
<td>2580</td>
<td>40</td>
<td>24</td>
<td></td>
<td>1.3</td>
<td></td>
<td>—</td>
</tr>
<tr>
<td>Severence Hall, Cleveland</td>
<td>1930</td>
<td>15,700</td>
<td>1395</td>
<td>1890</td>
<td>20</td>
<td>13</td>
<td></td>
<td>1.7</td>
<td>1.6</td>
<td></td>
</tr>
<tr>
<td>Carnegie Hall, New York</td>
<td>1891</td>
<td>24,250</td>
<td>1985</td>
<td>2760</td>
<td>23</td>
<td>16</td>
<td></td>
<td>1.8</td>
<td>1.8</td>
<td>1.6</td>
</tr>
<tr>
<td>Opera House, San Francisco</td>
<td>1932</td>
<td>21,800</td>
<td>2165</td>
<td>3250</td>
<td>51</td>
<td>30</td>
<td></td>
<td>1.7</td>
<td></td>
<td>—</td>
</tr>
<tr>
<td>Arie Crown Theatre, Chicago</td>
<td>1961</td>
<td>36,500</td>
<td>3265</td>
<td>5080</td>
<td>36</td>
<td>14</td>
<td></td>
<td>2.2</td>
<td>1.7</td>
<td>1.4</td>
</tr>
<tr>
<td>Royal Festival Hall, London</td>
<td>1951</td>
<td>22,000</td>
<td>2145</td>
<td>3000</td>
<td>34</td>
<td>14</td>
<td></td>
<td>1.5</td>
<td>1.5</td>
<td>1.4</td>
</tr>
<tr>
<td>Royal Albert Hall, London</td>
<td>1871</td>
<td>86,600</td>
<td>3715</td>
<td>6080</td>
<td>65</td>
<td>70</td>
<td></td>
<td>2.6</td>
<td>2.2</td>
<td></td>
</tr>
<tr>
<td>Concertgebouw, Amsterdam</td>
<td>1887</td>
<td>18,700</td>
<td>1285</td>
<td>2200</td>
<td>21</td>
<td>9</td>
<td></td>
<td>2.2</td>
<td>2.1</td>
<td>1.8</td>
</tr>
<tr>
<td>Kennedy Center, Washington</td>
<td>1971</td>
<td>19,800</td>
<td>1220</td>
<td>2760</td>
<td>—</td>
<td>—</td>
<td></td>
<td>2.5</td>
<td>2.2</td>
<td>1.9</td>
</tr>
</tbody>
</table>

CHAPTER 24

SOUND SOURCES IN A ROOM

POWER IN WATTS

\[ L_W = 10 \cdot \log_{10} \left(\frac{W}{W_0}\right) \]

speaking

\[ W = 10^{-5} \text{watts} \quad L_W = 10 \cdot \log_{10} \left(\frac{10^{-5}}{10^{-7}}\right) = 10 \cdot \log_{10} 10^2 = 20 \text{ dB} \]

HW) FIND THE ENERGY TO:

RAISE AN 8 OUNCE CUP OF WATER - 0°C TO 100°C.

\[ \text{C: SPECIFIC HEAT: 1 CAL/GRAM PER °C} \]

\[ 1 \text{ CAL} = 4.186 \text{ JOULES}, \quad 1 \text{ WATT} = 1 \text{ JOULE/SEC} \]

\[ 1 \text{ OUNCE} = 28.35 \text{ GRAMS} \]

USE: \[ \bar{Q} = C \cdot M \cdot \Delta T \]

\[ \bar{Q} = \text{HEAT ADDED} \]

in years

A) HOW LONG WOULD YOU HAVE TO TALK AT 70 dB TO GENERATE THIS AMOUNT OF ENERGY Q?

B) HOW MANY SPEAKERS TALKING AT 70 dB EACH FOR 1 HOUR WOULD GENERATE THIS AMOUNT OF ENERGY Q?