LECTURE 14

INTERFERENCE DIFFRACTION CONTINUED

SUMMARY - INTERFERENCE

\[ OP_2 - OP_1 = \sqrt{L^2 + \left(y + \frac{D}{2}\right)^2} - \sqrt{L^2 + \left(y - \frac{D}{2}\right)^2} \]

\[ = n\lambda \quad \text{CONSTRUCTIVE} \quad n = 0, \frac{1}{2}, \frac{3}{2}, \ldots \]

\[ (m + \frac{1}{2})\lambda \quad \text{DESTRUCTIVE} \quad m = 0, 1, 2, \ldots \]

FRAUNHOFER LIMIT \( L \gg \left(y + \frac{D}{2}\right) \)

EXTRA DISTANCE = \( D \sin \theta = D \frac{y}{L} \)

CONSTRUCTIVE \( \frac{Dy}{L} = n\lambda \quad y = y_n \)

\[ \frac{Dy_n}{L} = n\lambda \]

\[ y_n = \frac{n\lambda L}{D} \quad \frac{1}{D} \quad \text{SMALLER D} \]

\[ y_n = \frac{n\lambda L}{D} \quad \frac{1}{D} \quad \text{LARGER Y} \]

\[ \frac{D}{D_1} \quad \left\{ D_2 < D_1 \right\} \]
INTERFERENCE / DIFFRACTION

SUMMARY

BIGGER $\lambda$ BIGGER $\gamma$

$\lambda \cdot \text{DSIN} \theta = \lambda_1$

$\nu \cdot \text{DSIN} \theta = \lambda_2$

BEND THE RAY MORE TO ACCOMODATE LARGER $\lambda$. 

$N$ SOURCES.

$\delta = \frac{2\pi}{\lambda_0}$

$\delta = \frac{2\pi}{\Delta \lambda}.

\delta = \text{DSIN} \theta.$

$1 + e^{i \delta} + e^{2i \delta} + \ldots + e^{i(N-1)\delta}$

$1 = \frac{A^{1/2}}{\sin \frac{\delta}{2}}$

$|R_1| = A \left| \frac{\sin \frac{N\delta}{2}}{\sin \frac{\delta}{2}} \right|$

$A^2 \frac{N^2}{\delta^2} = \frac{(AN)^2}{\frac{2}{\pi^2}} \frac{2}{\pi^2} \frac{(AN)^2}{\frac{2}{\pi^2}}$

$\delta = 0$

$\delta \rightarrow$

$\delta_1 = \pi \rightarrow$
INTERFERENCE

ALGEBRAIC METHOD N-SLICE

\[1 + e^{is} + e^{2is} + \ldots + e^{(N-1)is}\]

\[x = e^{is}\]

\[1 + x + x^2 + \ldots + x^{N-1}\]

NOTE \(1 + x + x^2 + \ldots + x^\infty = \frac{1}{1-x} \text{ Multiply by } (1-x)\)

\[1 + x + x^2 + \ldots + x^\infty \]

\[\frac{1}{1-x} \cdot \frac{1}{1-x} \text{ (1-x)}\]

\[1 + x + x^2 + \ldots + x^\infty \]

\[-x - x^2 - \ldots - x^\infty\]

\[1 + 0 + 0 = 0\]

WRITE \((1 + x + \ldots + x^{N-1}) = (1 + x + \ldots + x^\infty) - (x + x^2 + \ldots + x^N)\)

\[\frac{1}{1-x} \cdot \frac{1}{1-x} (1-x)\]

\[= \frac{1-x^N}{1-x} = \frac{1 - e^{in\delta}}{1 - e^{i\delta}} = \frac{e^{in\delta/2} (e^{-in\delta/2} - e^{in\delta/2})}{e^{i\delta/2} (e^{-i\delta/2} - e^{i\delta/2})}

\[\sin \delta = e^{-i\delta} - e^{i\delta}\]

\[e^{-i\delta} (e^{in\delta/2} - \frac{8 i n \delta}{\sin \delta/2})\]

\[= e^{i(n+1)\delta/2} \frac{\sin \delta}{\sin \delta/2}\]

MAGNITUDE \[\frac{\sin \delta/2}{\sin \delta/2}\]
The large peaks occur when the denominator in \( R \) which is \( \sin \frac{\delta}{2} \) is zero. The \( R = 0/0 \) but the answer is \( R = \frac{A}{N} \) since the vectors line up.

At \( \delta = 2\pi, 4\pi, \ldots \),

\[ \delta = \frac{2 \pi n}{\lambda} \]

\[ dsin \theta = \frac{2 \pi n \lambda}{\lambda} \]

\[ n = \text{position of principal maxima} \]

In a diffraction case of an opening, the initial behavior is similar but there is only one main peak.

This behavior does not repeat.
INTERFERENCE - 2 PLANE WAVES MOVING AT AN \( \theta \)
WITH RESPECT TO EACH OTHER.

\[
\begin{align*}
\text{Crest of 1} & \quad \text{Valley of 2} \\
\text{Crest of 2} & \quad \text{Valley of 1} \\
\end{align*}
\]

\[
\begin{align*}
\theta & = 90 - \theta \\
\sin \theta & = \frac{1/2}{AC} \\
AC & = \frac{1/2}{\sin \theta} \\
\text{Distance between max & zero} & \text{ Given AC, } \theta \text{ is determined.} \\
\text{Interference pattern of 1 & 2 is a} \\
\text{blues of bright and dark lines.} \\
\text{used in holography.} \\
\end{align*}
\]
Figure 13.50 (a) The creation of a transmission hologram of a toy locomotive. (b) Replay of a transmission hologram.

Scattered wave from object plus reference wave make an interference pattern on a film. Interference pattern contains complete information about object wave - amplitude of wave including its phase.
INTERFERENCE - (HOLOGRAPHY)

OBJECT EMITS WAVES AS A BUNCH OF POINT SOURCES WHICH INTERFERS WITH THE REFERENCE WAVE

PLANE WAVE

Etc.

CONSTRUCTIVE

D - DESTRUCTIVE

POINT SOURCE

LOOK AT FRINGES AT DISTANCE \( l \), \( b \gg \lambda \)

\[ \frac{y}{a_1} = \sqrt{(l + \frac{1}{2})^2 - l^2} = \sqrt{1 + \frac{4 \lambda}{\lambda^2} + \frac{\lambda^2}{4}} = \sqrt{1 + \frac{4 \lambda^2}{4 \lambda^2}} \]

\[ \approx \sqrt{1 + \frac{\lambda^2}{4 \lambda^2}} = \sqrt{1 + \frac{1}{4}} = \sqrt{1 + \frac{1}{4}} \]

\[ \frac{y}{b_2} = \sqrt{(l + \frac{3}{2})^2 - l^2} = \sqrt{3 + 3 \lambda^2 + \frac{9 \lambda^2}{4}} = \sqrt{3 + \frac{9 \lambda^2}{4}} \]

Etc.

GIVEN \( \lambda \) FROM THE FRINGE SPACING YOU CAN FIND \( l \) AND POSITION OF POINT SOURCE.

FRINGES ARE CIRCLES FOR POINT SOURCE \( E \) PLANE WAVE - SPHERE INTERSECTS PLANE TO MAKE CIRCLES
\[
\vec{R} = A + Be^{i(kx - \omega t)} \\
A = \sqrt{x^2 + y^2} \quad \frac{\partial}{\partial x} = \frac{\partial y}{\partial y}
\]

\[
|R|^2 = \left( (A + B \cos \theta) + iB \sin \theta \right)^2 = A^2 + B^2 + 2AB \cos \theta + B^2 \sin^2 \theta
\]

\[
|\vec{R}|^2 = A^2 + B^2 + 2AB \cos \theta + B^2 \sin^2 \theta
\]

\[
\sin^2 \theta + \cos^2 \theta = 1 \\
\theta = \frac{2\pi}{2}
\]

Conventional photography just measures \( |B|^2 \), the object wave.

A holographic photo uses the interference pattern of a reference wave & object wave to retain information about the phase of the object wave.

\[
\omega = \frac{\Delta \lambda}{\lambda} = \frac{\beta}{\lambda} \\
d = \frac{\omega \Delta x}{\lambda}
\]

See figure for \(|R|^2\) from Mathematica.
Interference Pattern

Source Amplitude \( A \)

Plane Wave Amplitude \( B \)

\[
\text{int}[A, B, D, y, x] = (A^2 + B^2) + 2AB \cos \left[ \frac{2\pi \sqrt{2Dy + y^2}}{x} \right]
\]

\[ D = \frac{\lambda}{\text{sin} \theta} \]

\( \lambda = 2 \)

\( A = 2 \)
\( B = 1 \)
\( D = 10 \)

\[ \text{Plot}[\text{int}[2, 1, 10, y, 2.], \{y, 0, 5\}] \]

\[ \text{Plot}[\text{int}[1, 1, 10, y, 2.], \{y, 0, 5\}] \]

\[ \text{Plot}[\text{int}[2, 1, 10, y, .5], \{y, 0, 5\}] \]
Huygens Principle & Snell's Law

**Picture I**

TIME $t = 0$ (A) intersects boundary and a spherical wave is emitted which travels at $u_e$.

At time $t = \frac{d_{BB'}}{u_e}$, (B) $\rightarrow$ (B') and B' now emits a spherical wave.

**Picture II**

$A \rightarrow A'$ which is a distance:

$$d_{AA'} = \frac{u_e t}{u_e} = \frac{d_{BB'}}{u_e}$$

**Picture I**: $\sin \theta_1 = \frac{d_{BB'}}{d_{AA'}} = \frac{d_{BB'}}{r}$

**Picture II**: $\sin \theta_2 = \frac{d_{AA'}}{l} = \frac{d_{AA'}}{r}$

From 1:

$$\frac{\sin \theta_1}{u_e} = \frac{\sin \theta_2}{u_e}$$
Huygens Spherical Wave

New Wave Front

This wave front has point sources on it that generate new spherical waves. The tangent to these waves is the new spherical wave front.
**Diffraction**

Many source interference → continuous limit

Diffraction has played an important role in the history of physics & biology.

1) Electron diffraction confirmed wave aspects of particles.

2) Diffraction was important in the discovery of the double helix in DNA.

In acoustics is diffraction important in sound emitted by instruments? - No.

Is diffraction important in room acoustics? Depends on λ & obstacles.

Interference - each small opening in the source of a wave

Huygens:

Each point on a wavefront is the source of a spherical wave.

New wave front (draw tangent to the emitted circles)
Huygens Construction and Instruments

Point sources for new wavefront

Instrument such as a trumpet

Huygens Construction and Obstacles

Opening

Interference pattern at distance L

L large
Fraunhofer limit

Three quantities λ, W, L

Diffraction important when

λ ~ W if λ < W classical limit

L >> λ, W Fraunhofer limit — picture above

L ~ λW Fresnel limit
The diffraction result is the limit \( N \to \infty, D \to 0 \) of the interference result for \( N \)-slits.

\[
\mathcal{D} \downarrow ~ \mathcal{D} \uparrow \quad N \to \infty \quad \frac{N}{D} \to W = \text{width of opening}
\]

\( A \) is amplitude for 1 outgoing wave needed \( NA \) finite for \( N \to \infty \)

\[
R = A \frac{\sin N \delta/2}{\sin \delta/2} \quad \delta = \frac{\lambda DA}{2DS \sin \theta} = \frac{2\pi DS \sin \theta}{\lambda}
\]

\[
\frac{N \delta}{2} = \frac{\pi (HD) \sin \theta}{\lambda} = \frac{\pi W \sin \theta}{\lambda}
\]

\[
\frac{\delta}{2} = \frac{1}{N} \frac{N \delta}{2} = \frac{1}{N} \left( \frac{\pi W \sin \theta}{\lambda} \right)
\]

\[
R = A^2 \frac{\sin^2 N \delta}{2} = A^2 \frac{\sin^2 \left( \frac{\pi W \sin \theta}{\lambda} \right)}{\sin \delta/2} = \frac{\sin^2 \left( \frac{\pi W \sin \theta}{\lambda} \right)}{N \lambda}
\]

\[
\sin x \approx x, \quad \sin^2 \left( \frac{\pi W \sin \theta}{\lambda} \right) \approx \left( \frac{\pi W \sin \theta}{\lambda} \right)^2
\]

\[
I = \text{Intensity} = \left( NA \right)^2 \frac{\sin^2 \left( \frac{\pi W \sin \theta}{\lambda} \right)}{N \lambda}
\]

\[
I_0 = \left( NA \right)^2 \quad \frac{\left( \frac{\pi W \sin \theta}{\lambda} \right)^2}{\left( \frac{\pi W \sin \theta}{\lambda} \right)^2}
\]

\[
\sin^2 x \to 1 \quad \text{as} \quad x \to 0 \quad x \to 0 \quad \theta \to 0
\]

\[
I = I_0 \frac{\sin^2 \frac{\pi W \sin \theta}{\lambda}}{\frac{\pi W \sin \theta}{\lambda}^2} \quad I_0 = \text{central intensity}
\]

\[
x = \frac{\pi W \sin \theta}{\lambda}
\]
Diffraction

\[ I = I_0 \frac{\sin^2 \alpha}{\alpha^2} \quad \alpha = \frac{\pi}{\lambda} \sin \theta \]

\[ \sin \theta = \frac{y}{L} \quad \frac{2\lambda y}{2L} \]

Zeroes. \( \sin^2 \alpha = 0 \Rightarrow \sin \alpha = 0 \)

\( \alpha = \pi, 2\pi, 3\pi, \ldots \) not a zero of \( I \sim \sin^2 \alpha \)

\[ \frac{\pi}{\lambda} \sin \theta = \frac{n \pi}{2} \]

[\( n \) zeroes \( \sin \theta = \frac{n \lambda}{2} \) \( n = 1 \)]

\( \sin \theta = \frac{\lambda}{2} \) looks constructive

\( \sin \theta = \frac{\lambda}{2} \) looks destructive

It is destructive
DIFFRACTION

\[ W \sin \theta = \frac{\lambda}{2} \quad \text{i.e. } \frac{1}{2} \lambda \text{ in extra distance} \]

Every point source in upper half has a corresponding point in lower half out of phase with it.

\[ n = 2. \quad W \sin \theta = 2\lambda \quad \text{for next zero} \]

\[ \frac{W}{4} \sin \theta = \frac{\lambda}{2} \quad \text{Divide opening in 4 parts.} \]

4 regions

1. \( \frac{W}{4} \)
2. \( \frac{W}{4} \)
3. \( \frac{W}{4} \)
4. \( \frac{W}{4} \)

Extra distance = \( \frac{\lambda}{2} \)

Region 1 & 2 are out of phase by \( \frac{\lambda}{2} \)

Region 3 & 4 are out of phase by \( \frac{\lambda}{2} \) \( \Rightarrow \) zero in intensity
DIFFRACTION

BETWEEN 2 ZEROS IS A MAX
1ST ZERO \[ \frac{W}{2} \sin \theta = \frac{\lambda}{2} \]
2ND ZERO \[ \frac{W}{4} \sin \theta = \frac{\lambda}{2} \]

APPROX EXPRESSION (VERY GOOD)

DIVIDE INTO 3 REGIONS

\[ \frac{\lambda}{2} \]

1. & 2. CANCEL OUT

3. IS LEFT ALONE

EXPECT \( \approx \frac{1}{3} \) IN AMPLITUDE

TO BE PRESENT \( \epsilon^4 \)

\( \left( \frac{1}{3} \right)^2 \) IN INTENSITY APPROX \( \approx \frac{I}{9} \)

STRING MODEL
FROM W-SLIT

\[ AV \]

\[ AV = 2\pi r + 4\pi r = 3\pi r \]

\[ r = \frac{AV}{3\pi} \]

\[ R = \text{DIAMETER} = 2r \]

1 1/2 TIMES AROUND \( R = \frac{AV}{3 \left( \frac{\pi}{2} \right)} \)
DIFFRACTION

CIRCULAR OPENING.

\[ \Theta = \frac{\lambda}{D} \; \text{FOR 1st ZERO.} \]

SLIT

\[ \sin \Theta_M = 1.22 \frac{\lambda}{D} = 1.22 \frac{\lambda}{D/2} \approx \frac{\lambda}{D} \; \text{RADINS} \]

NOTE \( \sin \theta < 1 \)

\[ \Rightarrow \; 1.22 \frac{\lambda}{D} \leq 1 \]

PUT IN \( \frac{\lambda}{D} \)

\[ I = I(0) \left[ \frac{2J'(k a \sin \Theta)}{k a \sin \Theta} \right]^2 \]

\[ J(x) \sim \frac{x}{2} \quad J(x) \sim \frac{1}{x} \cos \left[ x - \frac{\pi}{2} - \frac{\pi}{4} \right] \]

1st ZERO OF \( J(x) \) IS AT \( x = 3.832 \)

2nd - 7.016 \; 3rd - 10.173

\[ k a \sin \Theta_M^1 = 3.832 = \frac{2\pi \frac{D}{2}}{\lambda} \cdot \frac{2\pi \frac{D}{2}}{\lambda} \]

\[ \sin \Theta_M^1 = \frac{3.832 \lambda}{\frac{D}{2}} = 12197 \frac{\lambda}{D} = 1.22 \frac{\lambda}{D} \]

2nd

\[ \sin \Theta_M^2 = \frac{7.016 \lambda}{\frac{D}{2}} = 2.23 \frac{\lambda}{D} \; \sin \Theta_M^2 = 10.173 \frac{\lambda}{D} \]

\[ = 3.24 \frac{\lambda}{D} \]
Diffraction
Circular
\[ x = \cos \theta \]

\[ I = I_0 \left( \frac{2J_1(x)}{x} \right)^2 \quad J = \frac{1}{\sqrt{\pi x}} \cos \left( x - \frac{\pi}{2} - \frac{\theta}{4} \right) \]

\[ \approx I_0 \frac{4 r_0}{\pi} \frac{1}{x^3} \cos^2 \left( x - \frac{\pi}{2} - \frac{\theta}{4} \right) \sim \frac{1}{x^3} \]

Approximate zeroes are zeroes of
\[ \cos \left( x - \frac{\pi}{2} - \frac{\theta}{4} \right) \]

\[ \cos x \]

\[ \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2} \]

\[ \frac{\pi}{2}, \frac{\pi}{2}, \frac{\pi}{2} \]

\[ \frac{\pi}{2}, \frac{\pi}{2}, \frac{\pi}{2} \]

\[ \frac{\pi}{2}, \frac{\pi}{2}, \frac{\pi}{2} \]

\[ \frac{\pi}{2}, \frac{\pi}{2}, \frac{\pi}{2} \]

\[ \frac{\pi}{2}, \frac{\pi}{2}, \frac{\pi}{2} \]

| \( x \) | \( \pi - \frac{\pi}{2} = \frac{\pi}{2} \) | \( \pi \) = \( \frac{\pi}{2} \) = 3.93 | 3.832 |
| \( x_1 \) | \( \frac{\pi}{2} \) | \( \frac{\pi}{2} \) = 3.93 | 3.832 |
| \( x_2 \) | \( \frac{\pi}{2} \) | \( \frac{\pi}{2} \) = 7.069 | 7.016 |
| \( x_3 \) | \( \frac{\pi}{2} \) | \( \frac{\pi}{2} \) = 10.01 | 10.17 |

Table exact
**DIFFRACTION**

**A4:** \( f = 440 \text{ Hz}, \quad v = 330 \text{ m/s} \)

\[ \lambda = \frac{330}{440} = \frac{3}{4} \text{ m} \]

Are these waves going to interfere - diffract?

\[ D \approx 5 \text{ cm} \]

\[ \sin \theta = 1.22 \frac{\lambda}{D} \approx \frac{\lambda}{D} \]

Need \( \lambda < D \) other \( \sin \theta > 1 \) which can't be.

\[ \frac{3}{4} \lambda > 5 \text{ cm} \quad \text{No diffraction} \]

3 octaves above \( f = 8.440 \text{ Hz} = 3520 \text{ Hz} \)

\[ A_7 \quad 2^3 \cdot 440 \text{ Hz} \]

\[ \lambda = \frac{330}{440} \cdot \frac{1}{4} = \frac{3}{4 \cdot 8} = \frac{3}{32} \approx \frac{1}{10} \text{ m} = 10 \text{ cm} \]

\[ D \approx 5 \text{ cm} \]

Still NO DIFFRACTION

\[ A_8 \quad \lambda = \frac{3}{4.16} \cdot 20 = 5 \text{ cm} \]

Still none.

**DOORWAY**

\[ \lambda = \frac{3}{4.16} \]

This is not circular.

\[ \min \text{ WS/NA} = \lambda \]

\[ \lambda = \sqrt{W} \]

It will spread out but NO MIN.

3 m

\[ \lambda = 0.75 \]
Diffraction from a rectangular opening

\[ \sin \theta_x = \frac{x}{L} \]
\[ \sin \theta_y = \frac{y}{L} \]

2D is a generalization of 1-D

**X-direction**
\[ I_x(\alpha) = I_x(0) \frac{\sin^2 \alpha}{\alpha^2} \]
\[ \alpha = \frac{\pi}{2} \frac{W_s \sin \theta_x}{x} \]
\[ = \frac{K W_s x}{2} \frac{x}{L} \]

**Y-direction**
\[ I_y(\beta) = I_y(0) \frac{\sin^2 \beta}{\beta^2} \]
\[ \beta = \frac{\pi}{2} \frac{W_s \sin \theta_y}{y} \]
\[ = \frac{K W_s y}{2} \frac{y}{L} \]

\[ I_{x'y'} = I_x(\alpha) I_y(\beta) \]
\[ = I(0) \frac{\sin^2 x \sin^2 y}{\alpha^2 \beta^2} \]
NON-LINEAR SYSTEMS

$\omega$, $2\omega$, $3\omega$ out also

NON-LINEAR SYSTEM

$\omega_1$, $\omega_2$, $\omega_3$

Ex. 7

$T = mg \cos \theta$

$-T = -lmg \sin \theta = I \frac{d^2 \theta}{dt^2}$

$-g \sin \theta = l \frac{d^2 \theta}{dt^2}$

Small $\theta$, $\sin \theta = \theta$, $-\frac{g}{l} \theta = \frac{d^2 \theta}{dt^2}$

$\theta = \theta_0 e^{i\omega t}$

$\theta_0$ is constant, $\theta_0 e^{i\omega t}$

$-\frac{g}{l} \theta_0 e^{i\omega t} = -\omega^2 \theta_0 e^{i\omega t}$

$\omega = \sqrt{\frac{g}{l}}$

$\sin \theta = \theta - \frac{\theta^3}{3!}$

$-\frac{g}{l} (\theta - \frac{\theta^3}{3!}) = \frac{d^2 \theta}{dt^2}$

Try $\theta_0 e^{i\omega t}$

$-\frac{g}{l} (\theta_0 e^{i\omega t} - \frac{\theta_0^3}{3!} e^{3i\omega t}) = -\omega^2 \theta_0 e^{i\omega t}$

No solution
SECTION 8.8 AURAL HARMONICS

SINGLE TONE AT f IF SUFFICIENTLY LOUD
CAN PRODUCE ADDITIONAL PITCHES AT 2f, 3f, ...
CALLED AURAL HARMONICS.

POWER LAW RESPONSE OF THE EAR
\[ x = a_0 + a_1 p + a_2 p^2 + a_3 p^3 + \ldots \]

p is sound pressure

a's determined experimentally.

\( p^2 \) generates the second harmonic of
the original pitch.

\( p^3 \) generates the third harmonic of
the original pitch.

**Diagram**

\[ A_i = \frac{a(e^{i\omega t} + e^{-i\omega t})}{2} + \frac{b(e^{i\omega t} + e^{-2i\omega t})}{2} \]

\[ A_i = a_0 + a_1 [A_i] + a_2 [A_i]^2 \]

same as incident
\[ [A_6]^2 = \frac{1}{2} \left( e^{i\omega_1 t} + e^{-i\omega_1 t} \right)^2 \]
\[ + \frac{1}{2} \left( e^{i\omega_2 t} + e^{-i\omega_2 t} \right)^2 \]
\[ + 2a \frac{b}{2} \left( e^{i\omega_1 t} + e^{-i\omega_1 t} \right) \left( e^{i\omega_2 t} + e^{-i\omega_2 t} \right) \]
\[ = \frac{a^2}{2} \left( e^{2i\omega_1 t} + e^{-2i\omega_1 t} + 2 \right)^2 + \frac{b^2}{2} \left( e^{2i\omega_2 t} + e^{-2i\omega_2 t} + 2 \right) \]
\[ + \frac{ab}{2} \left( e^{i(\omega_1 + \omega_2) t} + e^{i(\omega_1 - \omega_2) t} \right) + \frac{ab}{2} \left( e^{i(\omega_2 - \omega_1) t} + e^{i(\omega_2 + \omega_1) t} \right) \]

\( \omega_1 > \omega_2 \)

Cubic term \( [A_6]^3 = [A_6] [A_6]^2 \)

Will lead to more sidebands.