Homework 7.

1. Consider a circular membrane with radius $R=1$ m, and the wave speed $c=1000$ m/s. Write down the standing wave solutions $u_{mn}(r, \theta, t)$, and the corresponding frequencies $f_{mn}$ for

(a) $m=2$, $n=1$

(b) $m=2$, $n=3$

In your answers, substitute all the relevant numerical values. In particular, give the numerical values for $f_{mn}$ in Hz. You should obtain the approximate values for the Bessel’s function zeros $a_{mn}$ from the internet.

2. Sketch the nodal lines on the circular membrane of problem 1 for the cases (a) and (b).

3. As derived at lectures, the frequencies of a vibrating bar (transverse vibrations) of length $L$ clamped at one end are given by the approximate equation

$$f_{n\text{approx}} = \frac{\pi Kl_n^2}{8L^2},$$

where $n=1,3,5,7,...$ are odd integers. The lab manual “Vibration of plates & bars” (the lab done on March 8) gives the first three exact frequencies $f_{n\text{exact}}$ in terms of the same parameters.

(a) Calculate the ratio $f_{n\text{approx}}/f_{n\text{exact}}$ for the first three (lowest) frequencies. Is the approximate equation for $f_n$ getting better or worse as $n$ increases?

(b) Consider an aluminum bar with $L=0.2$ m, and thickness of 2 mm. The density and the Young’s modulus for aluminum are $2700$ kg/m$^3$, and $7.5 \times 10^{10}$ Pa, respectively. Calculate $f_{n\text{approx}}$, and $f_{n\text{exact}}$ for the first three frequencies.