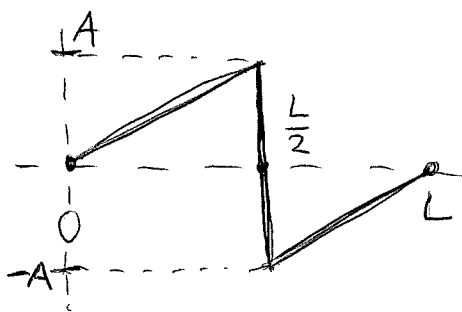


Homework 6

①



Consider a string of length L and speed of sound c with the initial shape $u(x, 0)$ shown at left.

All the segments are straight lines, the line at $x = \frac{L}{2}$ is vertical. The initial speed is zero at each x $\dot{u}(x, 0) = 0$.

(a) Argue that the Fourier coefficients

$$b_n = 0$$

(b) Do you expect ^{non-zero} coefficients a_n to be all odd, all even, or both even and odd?

(c) Calculate a_n for all n .

Write down the first three non-zero a_n explicitly.

② Consider the string from problem (1) with different initial conditions:

$$u(x, 0) = \psi(x) = 0 \quad (\text{the string is undisturbed initially})$$

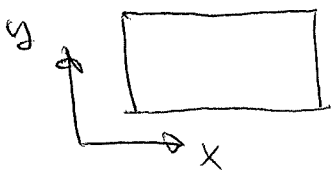
With $\dot{u}(x, 0)$ the same as shown in the plot of problem (1). In other words,

$$\dot{u}(x, 0) = \psi_{\text{problem 1}}(x)$$

Calculate the Fourier coefficients

a_n and b_n for all n .

③ Consider a rectangular membrane (all sides fixed) with $L_x = 2L$, $L_y = L$, and speed of sound v .



Sketch the membrane distortion for

(a) $n=1$ $m=1$ (fundamental mode)

(b) $n=3$ $m=1$

(c) $n=2$ $m=3$

And calculate the corresponding frequencies $f_{n,m}$