1) Due to Doppler effect, perceived frequency of echo is:

$$f_{echo} = f_0 \frac{v + v_t}{v - v_t} \tag{1}$$

where f_0 is the original frequency, v is sound velocity, v_t is speed of train. So:

$$f_{beat} = |f_{echo} - f_0| = f_0 \frac{2v_t}{v - v_t} \tag{2}$$

Solve v_t :

$$v_t = \frac{f_{beat} v}{2f_0 + f_{beat}} = 1.35m/s \tag{3}$$

2) Recall the damped, driven harmonic oscillator satisfies:

$$m\frac{d^2x}{dt^2} + \gamma m \frac{dx}{dt} + kx = F_0 cos(\omega t) \tag{4}$$

It has a steady-state solution with amplitude as function of driving frequency ω:

$$x(t) = \frac{F_0}{\sqrt{\omega^2\gamma^2 m^2 + (\omega^2 m - k)^2}} sin(\omega t - \phi) \tag{5}$$

and

$$A = \frac{F_0}{\sqrt{\omega^2\gamma^2 m^2 + (\omega^2 m - k)^2}} \tag{6}$$

a) $\omega \to 0$, $A \to F_0/k = 0.01m$
b) $\omega \to \infty$, $A \to 0$
c) Resonance occurs when amplitude A is maximal:

$$\frac{\partial}{\partial \omega}(\omega^2\gamma^2 m^2 + (\omega^2 m - k)^2) = 0 \tag{7}$$

So that

$$\omega_r = \sqrt{\frac{k}{m} - \frac{\gamma^2}{2}} \tag{8}$$

Substitute back:

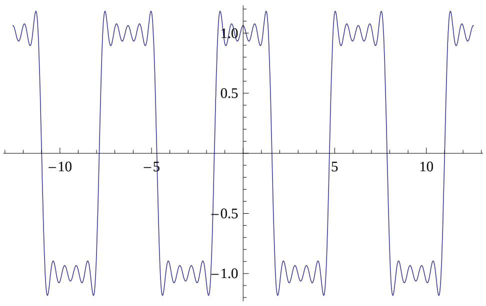
$$A = \frac{F_0}{\sqrt{k\gamma^2 m - \frac{\gamma^4 m^2}{4}}} = 100m \tag{9}$$

d)

$$Q = \sqrt{\frac{k}{\gamma^2 m}} = 10^4 \tag{10}$$

3) Note that $C_n = 0$ identically because the wave is symmetric under parity. (In the following diagrams, I have set $T = 2\pi$ for simplicity.)
a) $\alpha = 0.001$, The first 10 Fourier coefficient is:

$$\{1.27323, 0, -0.424388, 0, 0.254606, 0, -0.181833, 0, 0.141396, 0\} \tag{11}$$

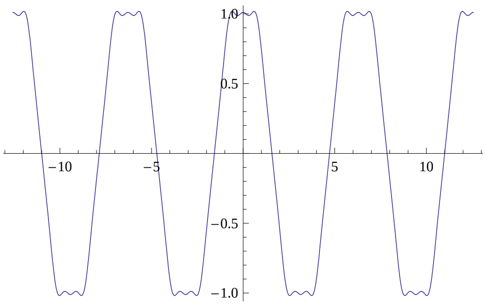


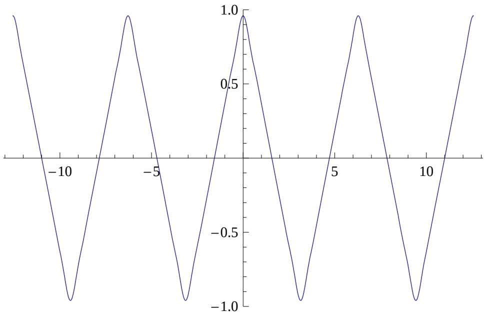
b) $\alpha = 0.125$:

$$\{1.14632, 0, -0.127369, 0, -0.0458527, 0, 0.0233943, 0, 0.0141521, 0\} \tag{12}$$

c) $\alpha = 0.250$:

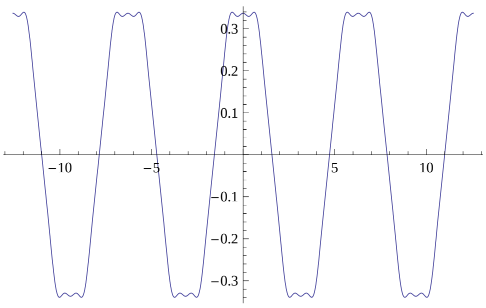
$$\{0.810569, 0, 0.0900633, 0, 0.0324228, 0, 0.0165422, 0, 0.010007, 0\} \tag{13}$$





d) $\alpha = 0.375$:

$$\{0.382106, 0, -0.0424562, 0, -0.0152842, 0, 0.00779808, 0, 0.00471736, 0\} \tag{14}$$



4)Recall that:

$$B_n = (\frac{2}{T}) \int_0^T dt f(t) cos(\frac{2\pi n t}{T}) \tag{15}$$

and

$$C_n = (\frac{2}{T}) \int_0^T dt f(t) sin(\frac{2\pi nt}{T}) \tag{16}$$

Plug $f(t) = \sum \delta(t - nT)$ in, one gets:

$$\begin{cases} B_n = \frac{2}{T} \\ C_n = 0 \end{cases} \tag{17}$$