Physics 301, Physics of Sound

Exam

1. Consider the three disturbances \( y(x,t) \) of a one-dimensional string running along the \( x \) axis:

\[
y(x,t) = 3e^{-(2x+3t)^2}
\]

\[
y(x,t) = \frac{2\cos(x^2 + 1)}{xt}
\]

\[
y(x,t) = \frac{1}{(x - 5t)^2 + 8}
\]

Indicate which of them describe a propagating disturbance (wave). For the propagating waves, find the speeds, and indicate the direction of propagation (positive or negative). All the distances are given in meters, and times in seconds.

2. A sinusoidal wave is given by the equation \( y(x,t) = 8 \sin(3x-5t) \). The distances are in meters, and time in seconds. Find: the speed of the wave \( v \), the wavelength \( \lambda \), the angular frequency \( \omega \), the frequency \( f \), and the amplitude \( A \).
3. An oscillator consists of mass $m=2$ kg connected to an ideal spring. The spring constant is $8$ N/m. A small damping (friction) of an unknown value is present in the system, and therefore the harmonic oscillator is damped.

(a) Calculate the approximate natural angular frequency $\omega_0$ of this system (the frequency in the absence of any external force).

(b) Now the system is driven by a sinusoidal external force with frequency $f_1=1.5$ Hz. What is the oscillation frequency of the system after a very long time passes?

(c) If the system is instead driven by a sinusoidal force with frequency $f_2=0.3$ Hz, will the amplitude of the resulting oscillations be larger or smaller than for the case described in (b)? Assume that a long time passes after the driving force is applied.

4. Consider two identical pendulums, connected with a spring. In the equilibrium position, the pendulums are vertical, and the spring is neither stretched, nor compressed, as shown in the figure.

(a) Draw the pictures showing the two natural oscillation modes of this system.

(b) The frequencies of the two natural modes are $5$ Hz and $5.5$ Hz. Indicate which frequency belongs to each of the two modes you’ve drawn. Explain your answer (a brief explanation will do).
5. A pipe is open at one end and closed at another. The length of the pipe is 10 cm, the speed of sound is 330 m/s. Calculate the fundamental frequency, and draw the corresponding distribution of the air pressure inside the pipe.

6. String one has length $L=1$ m, and the wave speed $v=2000$ m/s. The first harmonic (the next frequency after the fundamental) is excited in string one. String two has $L=0.5$ m and $v=2001$ m/s. It is excited at the fundamental frequency. The strings are fixed at the both ends.

(a) Calculate the frequencies of sound produced by string one and string two.

(b) What will you hear of the two strings produce the sound simultaneously?
7. A circular drumhead, fixed at the rim, may exhibit the two nodal patterns shown below. For each of these patterns, find the integers m and n for the corresponding standing wave solution \( u_{mn}(r, \theta, t) \). Which of these patterns correspond to a higher frequency? Explain your answer.

8. Calculate the speed of sound in helium and in air for \( T=300 \) K. The average molar mass of air is 29 g/mole, and of helium is 4 g/mole. Helium is a monoatomic gas with the adiabatic exponent \( \gamma = \frac{5}{3} \), and air is a diatomic gas with \( \gamma = \frac{7}{5} \). The gas constant R=8.3 J/mol K.
9. A sound wave is initially propagating in air. It then enters space filled with helium. The angle the sound wave makes with the line (dashed line) normal to the surface separating air from helium (solid line) is equal to $\theta=10^\circ$ in air, see the figure below. What is the angle with respect to the same normal for the sound wave after it crosses the air-helium boundary and propagates in helium? Note: the results of problem 8 will be useful.

10. A plane light wave of wavelength $\lambda=510$ nm is falling on a double slit (normal incidence). The slits are very narrow, the distance between the slits is 2 micrometers.

(a) Determine the angular positions if the first and the second diffraction maxima.

(b) Is it possible to observe the tenth maximum? How many maxima (on one side, not counting the central $n=0$ maximum) is it possible to observe?