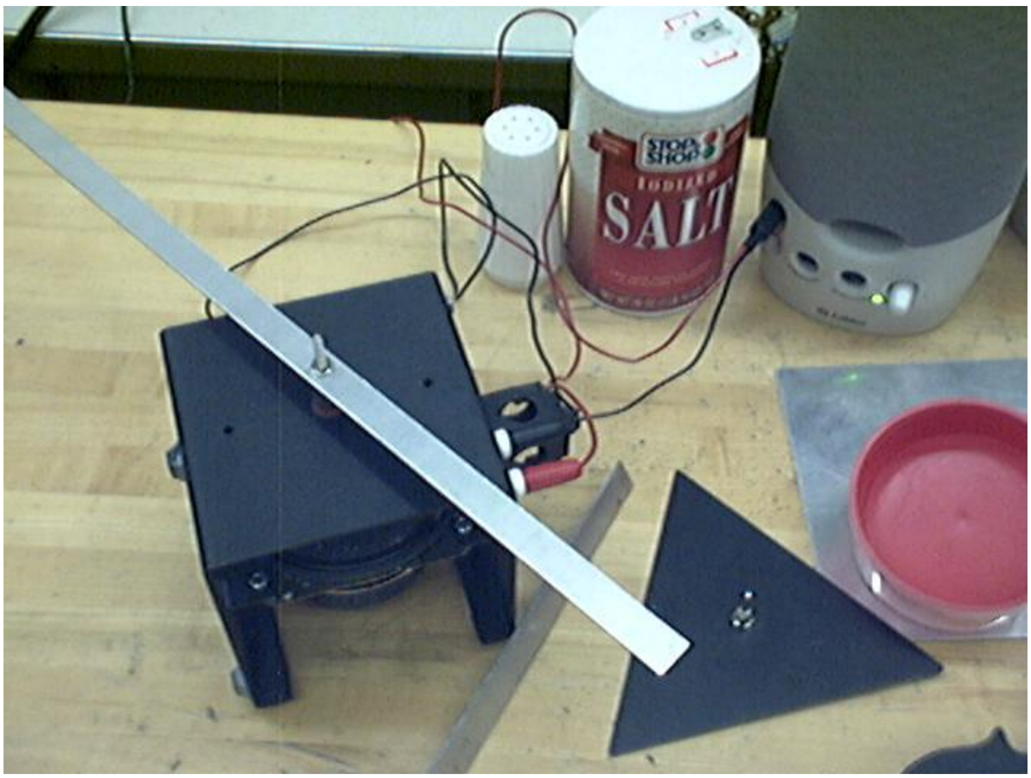
VIBRATION OF PLATES & BARS

The objectives of this experiment are:

- To observe the normal modes of a flat bar clamped at one end and free on the other.
- To observe the normal mode patterns of plates of different shapes using Chladni patterns.

APPARATUS: Function generator in FFTSCOPE, long and short flat bars, Chladni plates, salt, salt trays



INTRODUCTION

The equations of vibrational motion of plates involve fourth order spatial derivatives. For instance, bending waves in a long, thin bar or rod are described (to good approximation) by the equation:

$$\frac{\partial^2 y}{\partial t^2} = -\frac{YK^2}{\rho}\frac{\partial^4 y}{\partial x^4}$$

Solution of this equation with the boundary condition that the bar is clamped at one end and free on the other end gives normal mode frequencies expressed as:

$$f_1 = 0.56 \frac{K}{L^2} \sqrt{\frac{Y}{\rho}}$$

$$f_2 = 6.27 f_1$$

$$f_3 = 17.55 f_1$$

etc.

Where
y = transverse displacement of bar element
x = distance along length of bar
t = time
L = length of the bar
$K = h/\sqrt{12}$ = is the radius of gyration
h = thickness of the bar
Y = Young's modulus
ρ = mass density

In the case of aluminum, ρ = 2.7 x10^3 kg/m^3 and a typical (but can vary according to manufacturing process) value of Young's modulus is Y = 7.5 x 10^{10} N/m^2.

To solve for the normal modes of plates of other geometries, different fourth order equations must be solved. These are two dimensional problems: the solutions are functions of the y and x coordinates of positions on the plate. The long, thin one dimensional bar is a limiting case of such an equation. The solutions become very complicated for the free boundary condition at the perimeter of the plate. The Chladni patterns are very useful in visualizing the vibrational modes. When a normal mode is excited in the plate, small sand particles sprinkled onto the plate tend to collect at the nodal lines. These nodal line patterns were observed and studied by the 18th-century German physicist Ernst Chladni (pronounced KLAD-nee), and hence bear his name.

The sine wave output current of the function generator controlled by FFTSCOPE is connected to a mechanical vibrator. The vibration is induced by the Lorenz force between the current and a magnetic field. In other words, the alternating current passing through a small element of the wire experiences a force the direction of which alternates according to the equation $F = I(dl \times B)$. Note that the sign of the force depends on the sign of the current, which is determined by the direction of flow.

Bars and plates to be tested can be attached to the vibrator by simply inserting the plug into the receptacle at the top. When removing plates, hold the receptacle to prevent damage to the equipment.

Procedure:

I. Bar

Attach the long rectangular bar (about 50 cm length) aluminum plate to the vibrator. Measure the fundamental mode frequency of the longer side and then that of the shorter side. When you look for resonance in the longer side, you can hold the shorter side by hand to approximate the fixed boundary condition. Leave the other end free. When you look for modes in the shorted side, hold the longer side by hand. You can increase or decrease the frequency of the driving voltage by pressing up or down keys on the keyboard, respectively. Pressing them slowly will change the frequency by 0.1 Hz, while pressing them in quick succession will change it by 1 Hz. The page up and page down keys will have a similar effect. Pressing them slowly (rapidly) will change the frequency by 10 (100) Hz. Compare the measured fundamental frequency of the longer side with the formula given above. Do the same for the shorter side. The thickness is 1/16 inch. Measure the length with a ruler.

Look for higher normal modes and note the frequencies.

II. Plates

Change the set up slightly to study the vibration of plates. There are several shapes available. The plate should be made horizontal to minimize falling salt particles.

Start with a square plate. Change the frequency slowly to look for normal modes by watching the nodal patterns develop on sprinkled salt grains. You don’t need much salt to see these patterns take shape. Sketch three of these Chladni patterns and note the corresponding normal frequency. Plates with mounting holes located at different positions are available, so try them to see the effects of drive position.

Change the plate to rectangular, triangular, and violin shaped. Look for interesting patterns and record three of each on your hand-in sheet.

VIBRATION OF PLATES & BARS

LAB REPORT FORM

NAME: ____ DATE: ______________

PARTNER(S): ____
I.
Dimensions of the sample bars. Record the units.

longer bar

L(length) =____________________, h (thickness) = ___________________

Measured f_1 = ________________ ± ___________________

Calculated f_1 = _______________ using Y= 7.5 x 10^{10}• N/m^2

shorter bar

L(length) =____________________, h (thickness) = ___________________

Measured f_1 = ________________ ± ___________________

Calculated f_1 = _______________ using Y= 7.5 x 10^{10}• N/m^2

(Observed f_1 for shorted bar)/(Observed f_1 for longer bar) =____________

expected ratio based on the change in length = _____________________ (Note that resonant frequencies are proportional to L^{-2}.)

longer bar
Observed higher mode frequencies:

f_2 = ____________________ f_3 = _____________________

Expected higher mode frequencies:

f_2 = ____________________ f_3 =_____________________

Are the experimental resonant frequencies in accordance with the formulas? Discuss briefly and indicate what might have gone wrong if agreement is poor.

II. dimensions of the square plate: ________________________

Chladni patters and frequencies observed:

f = ______ f = ______ f = ______

dimensions of rectangular plate: _____________________

Chladni patterns and frequencies observed:

f = ______ f = ______ f = ______

dimensions of triangular plate:____________________

Chladni patterns and frequencies observed:

f = ______ f = ______ f = ______

Extra credit: Time permitting, draw three Chladni patterns for the violin-shaped plate below and give their frequencies.