Physics 301, Physics of Sound

Exam

Problem 1. Consider a disturbance $y(x,t)$ of an infinitely long one-dimensional string running along the $x$ axis. In this problem, all the distances are in meters, times in seconds, and speeds in meters/second.

(a) A wave is propagating in the negative $x$ direction. The speed of the wave is 3. For $t=0$, the measurement gives $y(x,0)=3e^{-2x^2}$. Write down the full equation $y(x,t)$ for this wave.

$$ y(x,t) = f(x-3t) = f(x+3t) \Rightarrow y(x,t) = 3e^{-2(x+3t)^2} $$

(b) A wave is propagating in the negative $x$ direction. The speed of the wave is 3. For $t=1$, the measurement gives $y(x,1)=3e^{-2x^2}$. Write down the full equation $y(x,t)$ for this wave.

$$ y(x,t) = f(x+3t) = 3e^{-2(x+3(t-1))^2} = 3e^{-2(x+3t-3)^2} $$

Problem 2. A sinusoidal wave $y(x,t)$ has amplitude 8 m, wavelength 3 m, and frequency 100 Hz. It is propagating in the positive $x$ direction.

(a) What is the speed of this wave?

$$ v = \frac{f}{\lambda} = 300 \text{ m/s} $$

(b) For $t=0$, $x=0$, one finds that $y(0,0)=0$. Write down the two possible equations $y(x,t)$ for this wave.

$$ y(x,t) = A \sin \left( \frac{2\pi}{\lambda} x - 2\pi f t + \varphi \right) $$

Two possibilities for $y(0,0)=0$ are $\varphi=0, \varphi=\pi$:

1) $y(x,t) = 8 \sin \left( \frac{2\pi}{3} x - 200 \pi t \right)$

2) $y(x,t) = -8 \sin \left( \frac{2\pi}{3} x - 200 \pi t \right)$
Problem 3. A one-dimensional oscillator consists of mass $m=1$ kg connected to a spring with spring constant $k=4$ N/m. There is a damping friction force $F=-bv$, where $v$ is the speed of the mass in m/s, and $b=8$ Ns/m.

(a) Which case better describes this oscillator: weak damping, or strong damping? Note: weak damping means that the system exhibits oscillatory motion (with decaying amplitude), no oscillatory motion is observed for the strong damping. Give a brief explanation.

$$\omega_0 = \sqrt{\frac{k}{m}} = 2 \text{ rad/s} \quad \beta = \frac{b}{2m} = 4 \text{ s}^{-1}$$

$\beta > \omega_0$ means **strong damping**

(b) The system described above is driven by a sinusoidal external force with period $T=600$ s. What is the angular frequency of the induced oscillations?

The same period. Therefore,

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{600 \text{s}} = 0.01 \text{ rad/s}$$

(c) The system described above is driven by a sinusoidal external force with period $T=600$ s. As time goes to infinity, which of the following values better describes the phase shift between the driving force and the induced oscillations? In radians: 0, $\pi/2$, $\pi$. Give a brief explanation. Hint: calculation of the natural frequency $\omega_0$ of this oscillator is useful.

Since $\omega < \omega_0$, the oscillations are in phase, and the phase shift is close to zero.
Problem 4.

A sound source is producing two sinusoidal waves with identical amplitudes and phases. Their frequencies are $f_1=1000$ Hz, and $f_2=1002$ Hz. The source is moving towards a stationary observer with the speed of 150 m/s. The speed of sound is 300 m/s. What is the frequency of the beats detected by the observer?

The detected Doppler-shifted frequencies are

$$f_1' = \frac{f_1}{1 - \frac{v}{c}} = \frac{1000}{1 - \frac{150}{300}} = 2000 \text{ Hz}$$

$$f_2' = \frac{f_2}{1 - \frac{v}{c}} = 2004 \text{ Hz}$$

$$f_{beats} = |f_1' - f_2'| = 4 \text{ Hz}$$

Problem 5. A pipe is closed at one end, and filled with helium gas in which the speed of sound is $c_{He}=1020$ m/s. The fundamental frequency produced by this pipe is 510 Hz. What would be the first harmonic frequency (the next frequency after the fundamental) produced by the same pipe when it is filled with air in which the speed of sound is $c=350$ m/s?

$$f_n = \frac{5}{4L} \cdot (2n+1) \quad n=0,1,2,3,\ldots$$

$$f_{He} = \frac{c_{He}}{4L}$$

$$L = \frac{c_{He}}{4f_{He}} = \frac{1020}{4 \cdot 510} = 0.5 \text{ m}$$

(n=0)

$$f_{air} = \frac{c}{4L} \cdot 3 = \frac{350}{4 \cdot 0.5} \cdot 3 = 525 \text{ Hz}$$

(n=1)
Problem 6. A point source of sound produces sound level of 60 dB one meter away from the source. Consider three identical sound sources of this kind at the same location, operating simultaneously. What is the sound level produced by these sources one meter away from them?

\[
\beta_1 = 10 \log \left( \frac{E}{E_0} \right) \quad \beta_3 = 10 \log \left( \frac{3E}{E_0} \right)
\]

\[
\beta_3 - \beta_1 = 10 \log 3 = 4.8
\]

Thus, \( \beta_3 = 64.8 \) dB

Problem 7. The fundamental (lowest) frequency of a vibrating square membrane is 1200 Hz. What is the vibration frequency of the same membrane for the mode with two nodal lines shown in the plot below?

\[
f_{m,n} = \frac{5}{2L} \sqrt{n^2 + m^2}
\]

The fundamental \( f_{1,1} = \frac{5}{2L} \sqrt{2} \),

\[
f_{2,1} = \frac{5}{2L} \sqrt{3} \quad \frac{f_{2,1}}{f_{1,1}} = \sqrt{3} = 2
\]

Thus, \( f_{2,1} = 2400 \) Hz
Problem 8. Consider a system with three coupled harmonic oscillators in one dimension. The system is described completely: you know all the masses, spring constants, spatial configuration, etc.

(a) In general, how many natural frequencies does the system have?

3.

(b) In general, how many initial conditions do you need to specify in order to determine the subsequent motion completely? Give an example of such initial conditions.

6.

For example, the initial positions and speeds of all the three masses.