GEOMETRIC OPTICS

PURPOSE: To review the basics of geometric optics and to observe the function of some simple and compound optical devices.

APPARATUS: Optical bench, lenses, mirror, target & light source, meter stick

INTRODUCTION: Optical instruments serve many functions. Microscopes can magnify an image to make extremely small objects visible. While telescopes can collect and concentrate light from very faint and distant objects. Lenses in photographic and video cameras form images for recording. Geometric optics ignores the true electromagnetic wave nature of light and assumes instead that light travels in a straight lines called a ray whose path through an optical instrument can be calculated with simple rules to predict the location of an image. In the next few pages we will review these rules.

Object-Image Relations for Lenses -- In geometrical optics we can get a good intuitive feeling for the behavior of lenses using these basic concepts:

1. All rays incident on a converging lens parallel to its axis are refracted in such a way that they cross the axis at a common point called the focal point or focus of the lens. The distance from the lens center to the focus is called the focal length, f. Each lens has two focal points.

2. After refraction by the lens, rays coming from an object will either converge to form a real image (as in a camera) or diverge so that they appear to come from a virtual image (as in a magnifying glass).

3. If the distance from the object to the lens is called s and the distance from the lens to the image s' (where s' is positive for a real image and negative for a virtual image) then the "thin lens" equation holds:

\[
\frac{1}{f} = \frac{1}{s} + \frac{1}{s'}
\]

The relationship between object, real image and focal length for a converging lens is illustrated in Fig 1. Two rays are drawn from the object to the lens. One ray is incident parallel to the axis and is refracted toward the focus, f. The other ray passes through the center of the lens, and will not be deviated because the surfaces of a "thin" lens are essentially parallel at the center. Using this you can construct any optical system.

It may be seen from the figure that the ratio of the distance of the image from the axis to the distance from the object to the axis (called the lateral magnification, m) is equal to - (s'/s).

The magnification is positive for an upright image and negative for an inverted image. The thin-lens equation can be used to show that
\[ m = -\frac{s'}{s} = -\left(\frac{s-f}{f}\right) \quad (2) \]

A special case of the thin lens equation occurs when the object distance, \( s \), is very large \((s \gg s')\) compared to the image distance \( s' \). Under these circumstances the \((1/s)\) term will be small enough to be neglected and the thin lens equation reduces to

\[
\frac{1}{s} + \frac{1}{s'} \rightarrow \frac{1}{s'} = \frac{1}{f} \quad \text{or} \quad s' = f \quad \text{when} \ (s \gg s') \quad (3)
\]

**Figure 1**  Object-Image Relationships for a Converging Lens
We will use this special case to estimate the focal length of the lenses used in the experiment. In the table on the next page, we summarize the sign convention for $m$ and $s'$.

<table>
<thead>
<tr>
<th>Sign</th>
<th>$m$, magnification</th>
<th>$s'$, image distance</th>
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<tbody>
<tr>
<td>+</td>
<td>upright image</td>
<td>real image</td>
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</table>
Example 1: The converging lens and the simple magnifier

Case a): \( s > f \). Suppose \( f = (+) \) 20 centimeters for the converging lens of figure 1 above. Then, if the object distance \( s = +30 \) cm, the thin lens formula is

\[
1/s + 1/s' = 1/f = 1/(+30) + 1/s' = 1/(+20),
\]

\[
1/s' = 1/20 - 1/30 = 1/60 \quad \text{and} \quad s' = +60 \text{ cm},
\]

where the plus sign for \( s' \) means the image is real and on the other side of the lens from the object.

The lateral magnification is \( m = \text{height of image/height of object} = (h_i/h_o) \). In this case it is equal to \( -(s'/s) = -60/30 = -2 \), indicating an enlarged image. The minus sign for \( m \) means that the image is inverted. For a real image it is possible to place a screen at the focal plane and directly measure the height of the image.

Case b): \( s < f \). Suppose the object distance \( s = +15 \) cm for the converging lens of \( f = 20 \) cm. Then the thin lens equation becomes

\[
1/(+15) + 1/s' = 1/20 \quad \rightarrow \quad 1/s' = 1/20 - 1/15 = -1/60 \quad \text{and} \quad s' = -60 \text{ cm}.
\]

The - sign for \( s' \) means that the image is virtual (on the same side of the lens as the object, impossible to view or measure on a screen). The magnification is \( m = -s'/s = -(60)/(+20) = +3 \). The + sign for \( m \) here means that the image is upright. We see that converging lens can give both real and virtual images, depending on the location of the object relative to the focal point.

![Figure 2](image)

**Figure 2**  
Object-Image Relationship for a Diverging Lens

Example 2: The diverging lens

Suppose the focal length of a diverging lens is \( f = -20 \) cm. If the object is at \( s = +30 \), solving the lens equation gives \( s' = -12 \), the - sign again indicating a virtual image:

\[
1/(+30) + 1/s' = 1/(-20) \quad \rightarrow \quad 1/s' = -1/20 - 1/30 = -5/60 \quad \rightarrow \quad s' = -12.
\]

The magnification is \( m = -(s'/s) = +12/30 = +0.4 \), so the image is reduced and erect (see figure 2). A diverging lens by itself can produce only virtual images.
Object-Image Relations for Mirrors -- Once you understand refractive optics, reflective optics are easy. In this case both object and image distances (s and s', respectively) are measured on the same side of the reflective surface. The math and equations are the same as with lenses. Consider the concave mirror in Fig. 3. Note that the radius of curvature, R, is not the focal length. A point source of light placed at the center of R would have the light focused back on itself.

![Diagram of Object-Image Relationship for a Concave Mirror]

**Figure 3** Object-Image Relationship for a Concave Mirror
Refracting telescopes

Astronomical Telescope

Two converging lenses are placed at a separation $f_1 + f_2$. $f_1$ is the focal length of the lens which collects light from the distant object (objective lens) and produces a real, inverted image, which is the object for lens 2. The function of lens 2 (ocular or eyepiece) is to produce a magnified virtual image (still inverted) for convenient viewing. The eyepiece acts like a simple magnifying lens. The magnification is $f_{\text{objective}}/f_{\text{eyepiece}}$.

A converging ("positive") objective lens ($f_1$) and a diverging ("negative") ($f_2$) ocular are used to make a short telescope (Figure 6, Terrestrial telescope) with an upright image. The lenses are spaced a distance approximately $f_1 + f_2$ apart (note that $f_2$ is negative since the lens is concave) and give an angular magnification of $-(f_1/f_2)$.

Terrestrial or Galilean Telescope

Figure 5

Figure 6
These telescopes are compound instruments consisting of an objective which has the function of gathering light and producing a real image. The second lens produces a virtual image of its object (the image formed by the objective). The eyepiece can be either converging ($s < f$) or diverging (always produces a virtual image).

**PROCEDURE:**
1. For lenses A, B, and C estimate their focal lengths by viewing a distant object.
2. For lenses A and C study and measure the relation between the image and object distances.
3. Use lens B to determine the negative focal length of lens D.
4. Make the same determination as step 2 above, but use a converging mirror. The special "bar" target must be inserted between the lamp (object) and the mirror. The image is very small and distorts easily if your alignment is poor.
5. Construct an Astronomical and a Terrestrial (Galilean) telescope. For the astronomical and Galilean telescopes start with lens separation of $(f_{\text{objective}} + f_{\text{eyepiece}})$ as in the diagrams, but remember that, in the Galilean telescope, the eyepiece is a diverging lens (this gives an erect image, in contrast to the astronomical telescope) and its $f$ is negative. ("Right side up" or "upside down" is not meaningful for a star or galaxy.) For the astronomical telescope use lens C as the objective and lens A as the ocular. For the Galilean telescope use lens C as the objective and lens D as ocular.

Adjust your telescope to focus on a distant eye chart or other object. Record the lens separation and give the ratio to the theoretical separation ($f_{\text{obj}} + f_{\text{eye}}$) for an object at infinity.

**How to Estimate Magnification:** Measuring magnification is not easy, but here is a hint. Use binocular vision; that is, keep both eyes open when looking through your telescope. Your mind will overlap the images which you can mentally compare. The wall blocks or a piece of paper on the wall serve as nice targets.

6. Optional: Use the simulation program **OPTICS LAB** to study the lens makers formula:

$$\frac{1}{f} = (n - 1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right)$$

**OPTICS LAB** can be found on the computers in room 101. A tutorial to follow will be handed out in class.
### Lenses

#### Case 1: Estimating focal length of lenses

View distant object such as ceiling lights and estimate the focal length with a meter stick.

Object viewed: _______________ Approximate distance from lens __________

Focal lengths: A _______ B _______ C _______

#### Case 2: Measuring the focal length

For each lens measure the image distance $s'$ for three different object distances and calculate the average focal length. Remember to record the image size as well. For the third measurement choose so that $s = s'$.

**Lens: A**

<table>
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<tr>
<th>#</th>
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<th>$s'$</th>
<th>$f$</th>
<th>Average $f$</th>
<th>$h_o$</th>
<th>$h_i$</th>
<th>$m = \frac{h_i}{h_o}$</th>
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**Lens: B**

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For lens C measure the image distance s’ for three different object distances and calculate the average focal length. But, this time for the third measurement exchange the values of s with s’ that you used for the second measurement. [That is, make s for the third measurement equal to the s’ you found in the second measurement, and vise versa.] Warning: C has a long focal length, in some cases you will have to put the target on the table (off the optical bench)

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<thead>
<tr>
<th>#</th>
<th>s'</th>
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Ratio f (case 1) / f (case 2 average) = ____________.

While everything is focused, predict what will happen if you cover half of lens C. Carry out the experiment and describe what happens to the image.

In the third setup when s and s’ were exchanged, what happened with the magnification?

### Negative Lens

Put the target light source and screen ~60 cm apart. Put lens B ~30 cm from the target light source and move it until there is a sharp image on the screen. The screen is the position of the real image. Now, put lens D between the screen and lens B such that it is exactly 5 cm from the real image on the screen. The real image formed by B will now serve as the object for lens D. s becomes -5 cm for Eq. 1; the minus means the object for D is on the same side as its image. To observe the new image formed by D move the
screen back until you see a focused image. Measure the distance between lens D and the 
new image; this is $s'$. Calculate the focal length of lens D from Eq. 1.

\[
\frac{1}{f} = \frac{1}{s} + \frac{1}{s'}
\]

Focal Length of D: __________

How was the size of the image changed by adding lens D?

**Converging Mirror**

Put the ruled bar target ~20 cm from the light; the ruled side should face away from the 
light. Place the concave mirror facing the light such that it "looks through" the clear part 
of the bar target. Make the heights the same. Move the mirror to only a few centimeters 
away until a tiny image on the bar target is focused on the ruled bar. Try to record the 
image size which will be very small.

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What is $R$, the radius of curvature of the mirror? __________

**Telescopes**

**Astronomical Telescope**

Do not lift the optical benches; the holders and lenses will fall to the floor. Move them 
around on the table top to look across the room.

Objective lens C \quad $f_{obj} =$ ______

Ocular (eyepiece) lens A \quad $f_{oc} =$ ______
Lens separation, SEP = (f_{obj} + f_{oc}) Ratio: SEP/(f_{obj} + f_{oc}) = 

Estimate the magnification by visual inspection: use binocular vision (both eyes open: one looks through the scope, the other at the target). FR = focal ratio; m_{exp} = experimental magnification.

\[ m_{exp} = \frac{h(\text{image})}{h(\text{object})} \quad \text{FR} = -\frac{f_{obj}}{f_{oc}} \]

Ratio: \[ m_{exp} / \text{FR} \]

**Terrestrial (or Galilean) Telescope**

Do not lift the optical benches; the holders and lenses will fall to the floor.

Objective lens C \quad f_{obj} = 

Ocular (eyepiece) lens D \quad f_{oc} = 

Lens separation SEP = 

\[ (f_{obj} + f_{oc}) = \quad \text{Ratio: SEP}/(f_{obj} + f_{oc}) = \]

Estimate the magnification by visual inspection: use binocular vision (both eyes open: one looks through the scope, the other at the target).

\[ m_{exp} = \frac{h(\text{image})}{h(\text{object})} \quad \text{Focal Ratios (FR)} = -\frac{f_{obj}}{f_{oc}} = \]

Ratio: \[ m_{exp} / \text{FR} \]

What does the sign of the magnification indicate for these two telescopes? Why is it OK to use one for reading distant signs, while not the other?