BALLISTIC PENDULUM

INTRODUCTION: In this experiment you will use the principles of conservation of momentum and energy to determine the speed of a horizontally projected ball and use this speed to predict the distance the ball will travel before striking the ground, along with an estimate of the error in your prediction. You will then measure the distance the ball actually travels, determine the error in your measurement, and compare the predicted and experimental ranges to see if they agree.

EXPERIMENT: *Measuring the Projectile Speed* — Consider a steel ball of mass m that is fired from a spring-loaded gun into a catcher-swing of mass M. The ball has an initial velocity of v. The catcher is initially at rest and is free to swing like a pendulum. After capturing the ball, the catcher + ball have a velocity V. At the moment the ball is captured there is no net external force acting on the catcher + ball system. Thus, its linear momentum is constant, and from the law of conservation of momentum:

$$
\begin{aligned}
\text{initial momentum} &= \text{final momentum} \\
mv &= (m+M)V \\
\text{or} \qquad v &= \frac{(m+M)}{m}V. \qquad (1)
\end{aligned}
$$

The ball's velocity v can be computed if we can measure V, which should be easier since it is much slower ($M \gg m$). Note that kinetic energy is not conserved in this collision because of the dissipating force of friction which locks the ball into the catcher.

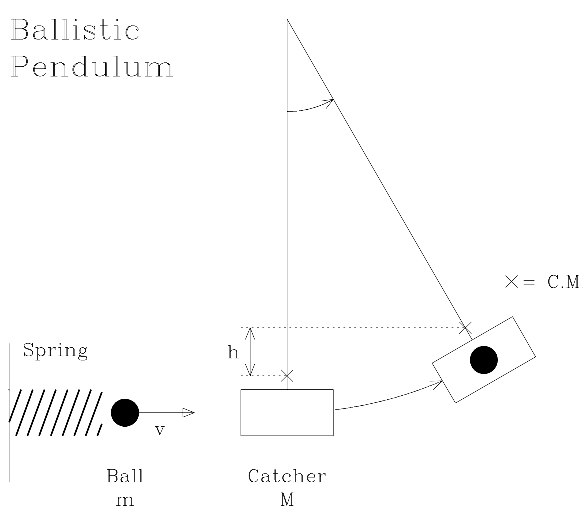

Figure 1: Ballistic Pendulum

After the ball is caught in the catcher and the swing-arm starts to move, momentum is no longer conserved because there is a net external force (the force of the swing-arm and gravity are no longer parallel). However, once the catcher-swing starts in motion, conservation of mechanical (kinetic plus potential) energy applies because the force of the swing-arm is always perpendicular to the motion of the catcher. As shown in Figure 1, the catcher with the ball continues to swing upwards until it stops with its center of mass at a vertical distance, h, above the starting level. The kinetic energy of the catcher plus ball, $K = 1/2(m+M)V^2$, has become gravitational potential energy, $(m+M)gh$.

¿From conservation of energy:

$$\frac{1}{2}(m+M)V^2 = (m+M)gh$$

$$V = \sqrt{2gh} \tag{2}$$

$$v = \frac{(m+M)}{m}\sqrt{2gh} \tag{3}$$

where eqn. (3) comes from combining eqns. (1) and (2).

A ratchet arrangement keeps the pendulum from falling back after it reaches its highest point and h is easily measured. [We assume that no energy is lost by drag of the ratchet latch as the catcher swings upward.] Thus the ball's initial velocity can be computed from easily measured quantities.

Calculation of the Range of the Projectile: We will now move the ballistic pendulum out of the way and fire the ball horizontally from an initial elevation y and see whether the ball hits the floor at the predicted horizontal distance x. See Figure 2.

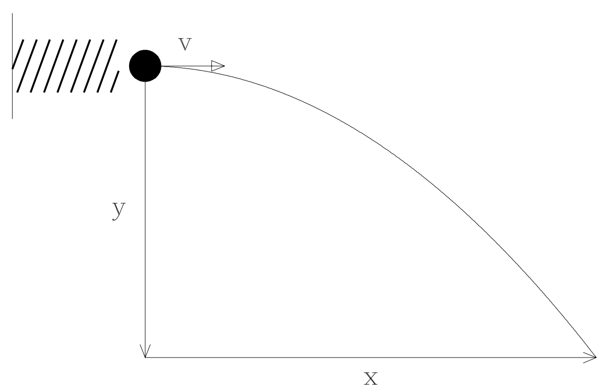

Figure 2: Range of Projectile

The horizontal component of the velocity is constant and equal to the initial speed v of the bullet so that

$$x = vt \tag{4}$$

where t is the time of flight of the bullet. Along the vertical, the bullet falls from rest with constant downward acceleration g so that

$$y = \frac{1}{2}gt^2. \tag{5}$$

Eliminating t between eqns. (4) and (5) gives

$$x = v\sqrt{2y/g} \tag{6}$$

Finally, combining eqns. (3) and (6) gives the location x at which the ball should hit the floor as predicted from a single pendulum measurement,

$$x = \frac{(m+M)}{m}\sqrt{2gh}\sqrt{2y/g}$$

which reduces to

$$x = \frac{(m+M)}{m}\,2\sqrt{hy} \tag{7}$$

Note that g cancels out. [Here is an important lesson: An analysis should becarried to the end using only symbols and mathematics. Don't use numerical quantities until you have to!] In order to make a prediction of the range x, you will measure h a number of times and use the average value, $\overline{h}$, in eqn. (7). You will then measure x a number of times and compare the average, $\overline{x}$, with the value of x from eqn. (7).

UNCERTAINTIES: *You should read and understand sections 2.8 and 2.9 in Baird.* In using eqn. (7) to predict where the projectile hits the floor we must take into account the errors in our measurements of the quantities we plug into the equation — m, M, h, and y. We will ignore the errors in measuring m and M since they are small. To get Δy, the error in y, you will simply estimate how accurate our measurement is — probably within the order ofa mm. To get Δh, the error in h, you will repeat the measurement a number of times and calculate the standard deviation, σ_h. Then $\Delta h = \sigma_h$. But we must now combine Δh and Δy to determine Δx. To do this we use the relation, that if x is a function f of y and h,

$$x = f(y,h)$$

Then

$$\Delta x = \frac{\partial f}{\partial y}\Delta y + \frac{\partial f}{\partial h}\Delta h,$$

where $\frac{\partial f}{\partial y}$ is the partial derivative of f with respect to y. All that is meant by the term partial derivative with respect to y is that you ignore that h is a variable and just take the regular derivative $\frac{df}{dy}$ (and similarly for $\frac{\partial f}{\partial h}$). Then from eqn. (7) we see

$$f(y,h) = \frac{(m+M)}{m}\,2\sqrt{hy}$$

and

$$\Delta x = \frac{(m+M)}{m}\left[\sqrt{\frac{h}{y}}\,\Delta y + \sqrt{\frac{y}{h}}\,\Delta h\right].$$

Dividing both sides of this equation by eqn. (7), we find the simple result:

$$\frac{\Delta x}{x} = \frac{\Delta y}{2y} + \frac{\Delta h}{2h}. \tag{8}$$

Thus, we predict the projectile will land at range values of $x + \Delta x$ and $x - \Delta x$, where x is given by eqn. (7) and Δx is given by eqn. (8).

PROCEDURE:

Part A When cocking the spring plunger, push straight back so as not to bend the rod. Careful: there are several catches on the plunger: always use the same catch. Shoot the ball into the catcher and measure the vertical rise h (measure to mark on catcher arm showing center of mass of combined system of catcher + ball). Repeat at least five times. The numbers on the ratchet are only for reference. Calculate $\overline{h}$, the average of your measurements, and the standard deviation, σ_h.

Measure the mass, m, of the ball with a balance to within 0.1 gm. The mass (M) of the catcher, is given to you on the body of apparatus. Aim the mechanism toward the wall which should have a protective sheet to absorb the impact of the bouncing ball. **BE CAREFUL OF YOURSELF AND OTHERS; DON'T PLAY GAMES WITH THE APPARATUS! DO NOT COCK THE GUN UNTIL YOU ARE READY TO FIRE!** Measure with the meter stick the vertical distance y through which the ball will fall while in flight.

Part B Tape a large sheet of paper to the floor to where a test shot landed. In some cases a piece of carbon paper can be placed on the sheet to give a better imprint; often this is unnecessary and will slow you down. You will compare the predicted (theoretical) values with your what you measure in the experiment.

SAFETY WARNING: Treat the spring-loaded gun with great caution. Load or cock the mechanism only just prior to firing. No one should ever be in front of a loaded gun. Warn others when you are ready to make a shot. Watch your fingers!