Lecture 18.

1. Maxwell-Boltzmann Energy distribution (see last lecture note)
   \[ P(E) = \frac{2}{N_{\text{vol}}} \beta^{3/2} E^{3/2} e^{-\beta E} \]
   \# of molecules @ E; \( N(E) \approx N \cdot P(E) \)
   
   Density of States: \( g(E) \propto E^{3/2} \) for quadratic degree of freedom.

2. Quantum Statistics.
   \( (\lambda \geq a) \).

   Wave functions of particles overlap. Need to consider quantum nature of microscopic particles, i.e. identical & indistinguishable.

   E.g., for 2 particles:
   \[ |\psi(1,2)|^2 = |\psi(2,1)|^2 \]
   \[ \Rightarrow \psi(1,2) = \pm \psi(2,1). \]

   
   \( " " : \text{Bosons} \quad \text{Bose-Einstein Stat.} \)

   \( " " : \text{Fermions} \quad \text{Fermi-Dirac Stat.} \)

   Anti-symmetric wave functions \( \rightarrow \) Pauli exclusion principle.

   Generalized to many particles case.

   # of particles allowed on A quantum states w/ energy \( E \):
   \[ n(E) = \left\{ \begin{array}{ll}
   0, 1 & \text{Fermions (no double occupancy),}
   0, 1, 2, \ldots & \text{Bosons (can be any #).}
   \end{array} \right. \]

   E.g., 4 particles in a 1D infinite well. (ignore spin) What's the lowest energy state?

   \[ n=4 \]
   \[ n=3 \]
   \[ n=2 \]
   \[ n=1 \]

   Fermion

   if \( N \) is large (e.g., \( N_{\text{Av}} \)).

   Recall \( PV = N \cdot 2 \langle kT \rangle \).

   Finite \( E_F \) \( \rightarrow \) finite pressure @ OK for \( E \) Fermi gas!

   \[ E \]
   \[ E \]
   \[ E \]

   Boson.

   Fermions

   Bosons

   Bose-Einstein Condensation (BEC)
Quantum text.

* Bose-Einstein distribution: \( f_{BE}(\varepsilon) = \frac{1}{e^{\beta(\varepsilon - \mu)} - 1} \) [\( \varepsilon \gg \mu \) \( f_{BE} \to 1 \)]

* Fermi-Dirac distribution: \( f_{FD}(\varepsilon) = \frac{1}{e^{\beta(\varepsilon - \mu)} + 1} \) [\( \varepsilon \ll \mu \) \( f_{FD} \to f_{MB} \)]

\( \mu \): chemical potential [tendency of particles to diffuse out of system]

Note: if \( \beta(\varepsilon - \mu) \) is large \( e^{\beta(\varepsilon - \mu)} \gg 1 \) \( \Rightarrow f_{BE} \approx f_{FD} \approx f_{MB} = e^{-\beta(\varepsilon - \mu)} \)

\( f_{MB} \) is a good approximation.

Fermions: \( e^- \), \( n \), \( p \), ...

Bosons: photons, phonons, Higgs, \( W^\pm \), \( Z \), ...

For metals (e.g. Cu, Ag, Au), \( \mu \sim 4-5\ eV \), \( kBT \sim 26\ meV \)

\( \log T < \mu \) i.e. zero T limit.

Degenerate Fermi gas (DFG)

The electronic properties are determined by \( e^- \) near \( E_F \) (more in Thermal Physics)

For photon gas (\( \mu = 0 \)), the energy density \( u(\varepsilon) \propto \frac{e^{\varepsilon/kT} - 1}{e^{\varepsilon/kT} + 1} \)

\( \rightarrow \) Planck formula of blackbody radiation.

(more in Thermal Physics)
**Bonds in molecules & Solid.**

1. **Ionic bond.** E.g. NaCl, \( \Theta \Theta \Theta \) electrostatic interaction.

2. **Covalent bond:** Molecules: \( \text{H}_2, \text{N}_2, \text{O}_2, \text{H}_2\text{O} \) sharing equivalent electrons.

3. **Metallic bond:** (Metals, e.g. Cu, Ag, Au) sharing mobile electrons.

4. **van der Waals bond:** Weak attraction between neutral molecules.

5. **Hydrogen bond:** Strong attraction between \( \text{H} \) in one molecule and an electronegative atom in another.

\[ \text{e.g. H}_2\text{O} \]

\[ \text{HF} \]

\[ \text{NH}_3 \]

**Molecular spectroscopy.** (rotation + vibration).

**Rotation.** \( E = \frac{l^2}{2I} \) \( l \): angular momentum

\( I \): moment of inertia \( I = \sum m_i r_i^2 \)

\( \alpha \text{M}. \) \( L^2 = \hbar (l+1) \)

\( \Rightarrow E_l = \frac{\hbar (l+1)}{2I} \)

*Typical energy scale:* \( \frac{\hbar^2}{2I} \sim \text{meV} \) microwave \( \sim \) Far IR.

**Vibration.** \( E_n = \hbar \omega \left( n + \frac{1}{2} \right) \)

\( \omega = \sqrt{\frac{k}{m}} \) \( m \): mass

\( \hbar \omega \sim 0.1 \text{ eV} \) \( \Rightarrow E_l \). Infrared.

**Energy spectrum can be studied by**

Optical spectroscopy (Infrared or Raman).

*Note: We will not cover 10.2-10.6 in this course.*