Chapter 7

30. For the 5\(d\) state \(n = 5\) and \(\ell = 2\). The possible \(m_\ell\) values are 0, \(\pm 1\), and \(\pm 2\), with
\[
m_\ell = \pm \frac{1}{2} \quad \text{for each possible } m_\ell \text{ value. The degeneracy of the } 5\!d \text{ state is then (with 2 spin states per } m_\ell \text{) equal to } 2(5) = 10.
\]

32. From problem 16, orbital degeneracy is \(n^2\), multiply spin degeneracy 2, the total degeneracy is \(2n^2\).

38. The radial probability distribution for the ground state is
\[
P(r) = r^2 |R(r)|^2 = \frac{4}{a_0^2} r^2 e^{-2r/a_0}.
\]

With \(r \ll a_0\) throughout this interval we can approximate the exponential term as \(e^{-2r/a_0} \approx 1\). Therefore the probability of being inside a radius \(1.2 \times 10^{-15}\) m is
\[
\int_0^{1.2 \times 10^{-15}} P(r)dr \approx \frac{4}{a_0^2} \int_0^{1.2 \times 10^{-15}} r^2 dr = \frac{4r^4}{3a_0^2} \bigg|_0^{1.2 \times 10^{-15}} = 1.55 \times 10^{-14}
\]

40. In general
\[
\langle r \rangle = \int_0^\infty r P(r)dr = \int_0^\infty r^3 |R(r)|^2 dr.
\]

For the 2\(s\) state:
\[
\langle r \rangle = \frac{1}{8a_0^3} \int_0^\infty r^3 \left(4 - \frac{4r}{a_0} + \frac{r^2}{a_0^2}\right) e^{-r/a_0} dr.
\]

\[
\langle r \rangle = \frac{1}{8a_0^3} \left[4(3!)a_0^4 - \frac{4}{a_0}(4!)(a_0^5) + \frac{1}{a_0^2}(5!)(a_0^6)\right] = \frac{a_0}{8} (24 - 96 + 120) = 6a_0
\]

For the 2\(p\) state:
\[
\langle r \rangle = \frac{1}{24a_0^3} \int_0^\infty r^3 \left(\frac{r}{a_0}\right)^2 e^{-r/a_0} dr = \frac{1}{24a_0^5} \int_0^\infty r^5 e^{-r/a_0} dr
\]
\[
= \frac{1}{24a_0^5} (5!)(a_0^6) = \frac{120a_0}{24} = 5a_0
\]
44. \( R = \frac{e^2}{4\pi\varepsilon_0 mc^2} = \frac{1.44 \times 10^{-9} \text{ eV} \cdot \text{m}}{511 \times 10^3 \text{ eV}} = 2.82 \times 10^{-15} \text{ m} \). From the angular momentum equation \( v = \frac{3\hbar}{4mcR} = \frac{3hc}{4mc^2R} c = \frac{3(197.33 \text{ eV} \cdot \text{nm})}{4(511 \times 10^3 \text{ eV})(2.82 \times 10^{-6} \text{ nm})} c = 103c \). A speed of 103c is prohibited by the postulates of relativity.

46. (a) The only change in Equation (7.3) is in the potential energy term, with \( V = -\frac{Ze^2}{4\pi\varepsilon_0 r} \) so the Schrödinger equation becomes:

\[
\frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial \psi}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial \psi}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 \psi}{\partial \phi^2} + \frac{2\mu}{\hbar^2} \left( E + \frac{Ze^2}{4\pi\varepsilon_0 r} \right) \psi = 0.
\]

(b) Because \( V \) occurs only in the radial part, there is no change in the separation of variables.

(c) Yes, from Equation (7.10) is it clear that the radial wave functions will change.

(d) No, there is no change in the \( \theta \) or \( \phi \) dependence.

50. The interaction between the magnetic moment of the proton and magnetic moment of the electron causes hyperfine splitting. The transition between the two states causes emission of a photon with energy of \( E = \frac{hc}{\lambda} = \frac{1240 \text{ eV} \cdot \text{nm}}{21 \times 10^7 \text{ nm}} = 5.9 \times 10^{-6} \text{ eV} \). From the uncertainty principle, we know \( \Delta E \Delta t \geq \frac{\hbar}{2} \). With a lifetime of \( \Delta t = 1 \times 10^7 \text{ y} \) then

\[
\Delta E \geq \frac{6.5821 \times 10^{-16} \text{ eV}}{2(1 \times 10^7 \text{ y})(3.16 \times 10^7 \text{ s/y})} = 1.041 \times 10^{-36} \text{ eV}.
\]
51. Break the vector \( \mathbf{r} \) into its components \( \mathbf{r} = xi + yj + zk \). Because the two factors \( R \) and \( Y \) in the hydrogen wave functions are given in spherical coordinates, it is best to express each of the Cartesian components in spherical coordinates using the standard transformations \( x = r \sin \theta \cos \phi \), \( y = r \sin \theta \sin \phi \), and \( z = r \cos \theta \) as shown in Figure 7.1. We know that in spherical coordinates, the volume element \( d\tau = r^2 \sin \theta drd\theta d\phi \).

In this case we have \( Y = Y_{00} = 1 / 2\sqrt{\pi} \) for both the initial and final states, so \( Y_i^* Y_f = \frac{1}{4\pi} \).

For simplicity we will write each component of the integral separately. Ignoring constant factors, the \( x \) component is \( \int \psi_f^* x \psi_i d\tau = \int_0^\pi \int_0^{2\pi} \int_0^r r^2 R_f^* R_i R d\tau \sin^2 \theta d\theta d\phi \).

The integral over \( \phi \) is zero, so the product is zero. Similarly, the \( y \) and \( z \) components of the integral are zero, and so the transition probability is zero. (For the \( z \) component, it is the integral over \( \theta \) that vanishes.)

Chapter 8

7. From Figure 8.4 we see that the radius of Na is about 0.16 nm. We know that for single-electron atoms \( E = -\frac{Z\epsilon^2}{8\pi\epsilon_0 r} \). Therefore

\[
Ze = -\frac{8\pi\epsilon_0 rE}{\epsilon^2} e = -2 \frac{4\pi\epsilon_0}{\epsilon^2} rEe = -\frac{2(0.16 \text{ nm})(-5.14 \text{ eV})}{1.44 \text{ eV} \cdot \text{nm}} e = 1.14e.
\]

12. \( J \) ranges from \( |L - S| \) to \( |L + S| \) or 2, 3, 4. Then in spectroscopic notation \(^{2S+1}L_J\), we have three possibilities: \(^3F_2\), \(^3F_3\), or \(^3F_4\). The ground state has the lowest \( J \) value, or \(^3F_2\).

With \( n = 4 \) the full notation is \(^4^3F_2\).

27. \( I = \frac{dq}{dt} = \frac{Ze}{2\pi r/v} = \frac{Ze v}{2\pi r} \) From the Biot-Savart Law, \( B = \frac{\mu_0 I}{2r} = \frac{\mu_0 Ze v}{4\pi r^2} \). With the assumption that the orbit is circular, the angular momentum is \( L = mvr \), so \( v = \frac{L}{mr} \) and

\[
B = \frac{\mu_0 Ze L}{4\pi mr^3} = \frac{Ze L}{4\pi \epsilon_0 mc^2 r^3} \]

where we have used the fact that \( \mu_0 = \frac{1}{c^2 \epsilon_0} \). The directions of the vectors follow from the right-hand rule.
28. Using the fact that $g = 2$ we have $\tilde{\mu}_s = g \frac{e\hat{S}}{2m} = \frac{e\hat{S}}{m}$ and thus $V_{\text{sl}} = \tilde{\mu}_s \cdot \tilde{B} = \frac{Ze^2}{4\pi\varepsilon_0} \frac{\hat{S} \cdot \hat{L}}{m^2 c^2 r^3}$.

29. (a) In order to use the result of the previous problem, we need to know the directions of $\hat{S}$ and $\hat{L}$. The electron has $S = \pm 1/2$, so $\|\hat{S}\| = \sqrt{3/4} \hbar$ and the angle $\hat{S}$ makes with the $+z$-axis is $\theta = \cos^{-1} \left( \pm 1/2 \right) = 54.7^\circ$ or $125.3^\circ$.

With $L = 1$ we have $\|\hat{L}\| = \sqrt{2} \hbar$ and the vector $\hat{L}$ can have three possible orientations, corresponding to $m_L = 0, \pm 1$. If we choose $m_L = 0$, then the orientation of the $\hat{L}$ vector is in the $xy$ plane. Therefore for either spin state the angle between $\hat{L}$ and $\hat{S}$ is $35.3^\circ$ and $\hat{S} \cdot \hat{L} = \frac{\sqrt{3}}{4} \hbar \sqrt{2} \hbar \cos(35.3^\circ) = \hbar^2$. Then using $r = 5a_0$ for a $2p$ electron (see Chapter 7 Problem 40), we have

$$V = \frac{e^2 \hbar^2}{4\pi\varepsilon_0 m^2 c^2 r^3} = \frac{(1.44 \text{ eV} \cdot \text{nm})(1240 \text{ eV} \cdot \text{nm})^2}{4\pi^2 \left( 5.11 \times 10^5 \text{ eV} \right)^2 \left( 5 \times 0.0529 \text{ nm} \right)^3} = 1.2 \times 10^{-5} \text{ eV}.$$ The difference between spin-up and spin-down states is twice this amount, or $2.4 \times 10^{-5} \text{ eV}$, which is just over half the measured value.

(b) The two possibilities are $j = 1/2$ and $j = 3/2$. The difference between these two is

$$\Delta V = \frac{Z^4 \alpha^4}{2n^3} mc^2 \left( \frac{2}{2(1/2)+1} - \frac{2}{2(3/2)+1} \right)$$

$$= \left( \frac{1}{137} \right)^4 \left( 5.11 \times 10^5 \text{ eV} \right) \left( \frac{2}{2(1/2)+1} - \frac{2}{2(3/2)+1} \right) = 4.53 \times 10^{-5} \text{ eV}$$

which gives a more accurate result.