PHY 273’ HW7 Solution

Ch7: 1, 2, 8, 16, 20, 21, 24.

1. Starting with Equation (7.7), let the electron move in a circle of radius \( a \) in the \( xy \) plane, so \( \sin \theta = \sin(\pi/2) = 1 \). With both \( r \) and \( \theta \) constant, \( R \) and \( f \) are also constant. Let \( R = f = 1 \). Then \( g = \psi \) and the derivatives of \( R \) and \( f \) are zero. With this Equation (7.7) reduces to \( -\frac{2\mu}{\hbar^2} a^2 (E - V) = \frac{1}{\psi} \frac{d^2\psi}{d\phi^2} \). In uniform circular motion with an inverse-square force, we know from the planetary model that \( E = V/2 \), and \( E - V = \frac{V}{2} = \frac{V}{2} = |E| \).
Thus \( -\frac{2\mu}{\hbar^2} a^2 |E| = \frac{1}{\psi} \frac{d^2\psi}{d\phi^2} \) or \( \frac{1}{a^2} \frac{d^2\psi}{d\phi^2} + \frac{2\mu}{\hbar^2} |E| = 0 \).

2. This is a simple harmonic oscillator equation. Assume a standard trial solution \( \psi = A \exp(iB\phi) \). With this trial solution \( d^2\psi / d\phi^2 = -B^2 \psi \). Substituting this into the equation from the previous problem we find: \( \frac{1}{a^2} (-B^2) \psi + \frac{2\mu}{\hbar^2} |E| \psi = 0 \). Solving for \( B \),

\[
B = \sqrt{\frac{2\mu |E| a}{\hbar}}.
\]

To find \( A \), normalize \( \int_0^{2\pi} \psi^* \psi d\phi = 1 = A^2 \int_0^{2\pi} d\phi = 2\pi A^2 \) so \( A = \sqrt{1/2\pi} \).

Note that \( B \) must be an integer (let \( B = n \)) so that \( \psi \) will be single-valued

\[ [\psi(0) = \psi(2\pi)] \]. With \( B = n \) we have \( n^2 = \frac{2\mu}{\hbar^2} |E| a^2 \) and \( |E| = \frac{n^2 \hbar^2}{2\mu a^2} \). For circular motion \( |E| = \frac{L^2}{2I} \) where rotational inertia \( I = \mu a^2 \) for a particle of mass \( \mu \). Thus

\[
L^2 = 2I |E| = 2\mu a^2 \frac{n^2 \hbar^2}{2\mu a^2} = n^2 \hbar^2 \text{ or } L = n\hbar \], which is the Bohr condition.
8. The wave function given is $\psi_{100}(r, \theta, \phi) = Ae^{-r/a_0}$ so $\psi^* \psi$ is given by $\psi_{100}^* \psi_{100} = A^2 e^{-2r/a_0}$.

To normalize the wave function, compute the triple integral over all space

$$\iiint \psi^* \psi \, dV = A^2 \int_0^{2\pi} \int_0^\pi \int_0^\infty r^2 \sin \theta e^{-2r/a_0} \, dr \, d\theta \, d\phi.$$  The $\phi$ integral yields $2\pi$, and the $\theta$ integral yields 2. This leaves $\iiint \psi^* \psi \, dV = 4\pi A^2 \int_0^\pi r^2 e^{-2r/a_0} \, dr = 4\pi A^2 \frac{2}{(2/a_0)^3} = \pi a_0^3 A^2$.

This integral must equal 1 due to normalization which leads to $\pi a_0^3 A^2 = 1$ so $A = \frac{1}{\sqrt{\pi a_0^3}}$.

16. There are $2l + 1$ states ($m_l$ values) for each $l$, and $l = 0, 1, \ldots, (n - 1)$, thus, the degeneracy of nth level is:

$$\sum_{l=0}^{n-1} (2l + 1) = 2 \sum_{l=0}^{n-1} l + n = 2 \times \frac{n(n-1)}{2} + n = n^2.$$  

20. The maximum difference is between the $m_l = -2$ and $m_l = +2$ levels, so $\Delta m_l = 4$. Then $\Delta V = \mu_B (\Delta m_l) B = (5.788 \times 10^{-5} \text{ eV/T})(4)(3.5 \text{ T}) = 8.10 \times 10^{-4} \text{ eV}$.

21. Differentiating $E = \frac{hc}{\lambda}$ we find: $dE = -\frac{hc}{\lambda^2} \, d\lambda$ or $|\Delta E| = \frac{hc}{\lambda^2} |\Delta \lambda|$. In the normal Zeeman effect, between adjacent $m_i$ states $|\Delta E| = \mu_B B$ so $\mu_B B = \left(\frac{hc}{\lambda_0^2}\right)|\Delta \lambda|$ or $\Delta \lambda = \frac{\lambda_0^2 \mu_B B}{hc}$.

24. From Problem 21, $\Delta \lambda = \frac{\lambda_0^2 \mu_B B}{hc}$ so the magnetic field is

$$B = \frac{hc\Delta \lambda}{\lambda_0^2 \mu_B} = \frac{(1240 \text{ eV} \cdot \text{nm})(0.04 \text{ nm})}{(656.5 \text{ nm})^2 (5.788 \times 10^{-5} \text{ eV/T})} = 1.99 \text{ T}.$$