Chapter 5

56. (a) These electrons have a kinetic energy that is a significant fraction of the rest energy, so they must be considered relativistically. These electrons have a total energy
\[ E = K + E_0 = 390 \text{ keV} + 511 \text{ keV} = 901 \text{ keV}. \]

\[ \lambda = \frac{h}{p} = \frac{hc}{pc} = \sqrt{E^2 - E_0^2} = \frac{1.240 \text{ keV} \cdot \text{nm}}{\sqrt{(901 \text{ keV})^2 - (511 \text{ keV})^2}} = 0.00167 \text{ nm} = 1.67 \text{ pm}. \]

(b) Using the Davisson-Germer result Equation (5.5),

\[ \sin \phi = \frac{n \lambda}{D} = \frac{(1)(0.00167 \text{ nm})}{0.215 \text{ nm}} = 0.07777 \text{ which gives an angle } \phi = 0.45^\circ. \]

This small angle is quite difficult to measure, which is why less energetic electrons are better for this type of scattering experiment.

62. (a) The uncertainty principle (Equation 5.45) provides the relationship between uncertainty in energy and time as, \( \Delta E \Delta t \geq \frac{\hbar}{2} \) or

\[ \Delta E \geq \frac{\hbar}{2(\Delta t)} = \left( 6.5821 \times 10^{-16} \text{ eV} \cdot \text{s} \right) / \left( 2 \times 8.4 \times 10^{-17} \text{ s} \right) = 3.92 \text{ eV}. \]

(b) The uncertainty in mass is given by the energy-mass relation as \( \Delta m = \frac{\Delta E}{c^2}. \)

With a mass of 135 MeV/c^2, the relative uncertainty is

\[ \frac{\Delta m}{m} = \frac{\Delta E}{mc^2} = \frac{3.92 \text{ eV}}{135 \times 10^6 \text{ eV}} = 2.9 \times 10^{-8}. \]

Chapter 6

5. \( \Psi^* \Psi = A^2 r^2 \exp \left( \frac{-2r}{\alpha} \right). \)

The condition for normalization becomes

\[ \int_0^\infty \Psi^* \Psi \, dr = A^2 \int_0^\infty r^2 \exp \left( \frac{-2r}{\alpha} \right) \, dr = A^2 \left[ \frac{2}{(2/\alpha)^3} \right] = \frac{A^2 \alpha^3}{4} = 1. \]

Therefore

\[ A = \sqrt{\frac{4}{\alpha^3}} = 2\alpha^{-3/2}. \]
10. Using the Euler relations between exponential and trig functions, we find
\[ \psi = A \left( e^{ix} + e^{-ix} \right) = 2A \cos(x). \]

Normalization: \[ \int_{-\pi}^{\pi} \psi^* \psi \, dx = 4A^2 \int_{-\pi}^{\pi} \cos^2(x) \, dx = 4A^2\pi = 1. \] Thus \( A = \frac{1}{2\sqrt{\pi}}. \)

(a) The probability of being in the interval \([0, \pi/8]\) is
\[ P = \int_{0}^{\pi/8} \psi^* \psi \, dx = \frac{1}{\pi} \int_{0}^{\pi/8} \cos^2(x) \, dx = \frac{1}{\pi} \left( \frac{x}{2} + \frac{1}{4} \sin(2x) \right) \bigg|_{0}^{\pi/8} \]
\[ = \frac{1}{16} + \frac{1}{4\pi\sqrt{2}} = 0.119. \]

(b) The probability of being in the interval \([0, \pi/4]\) is
\[ P = \int_{0}^{\pi/4} \psi^* \psi \, dx = \frac{1}{\pi} \int_{0}^{\pi/4} \cos^2(x) \, dx = \frac{1}{\pi} \left( \frac{x}{2} + \frac{1}{4} \sin(2x) \right) \bigg|_{0}^{\pi/4} \]
\[ = \frac{1}{8} + \frac{1}{4\pi} = 0.205. \]

12. \( E_n = \frac{n^2 \pi^2 h^2}{2mL^2}; \quad E_{n-1} = \frac{(n+1)^2 \pi^2 h^2}{2mL^2}; \)
\[ \Delta E_n = E_{n+1} - E_n = \frac{\pi^2 h^2}{2mL^2} \left[ (n+1)^2 - n^2 \right] = \frac{\pi^2 h^2}{2mL^2} (2n+1). \]

Computing specific values:
\[ \Delta E_1 = \frac{\pi^2 h^2}{2mL^2} (3); \quad \Delta E_8 = \frac{\pi^2 h^2}{2mL^2} (17); \quad \Delta E_{800} = \frac{\pi^2 h^2}{2mL^2} (1601) \]

14. (a) We know the energy values from Equation (6.35). The energy value \( E_n \) is proportional to \( n^2 \) where \( n \) is the quantum number. If the ground state energy is 4.3 eV ,
then the next three levels correspond to: \( 4E_1 = 17.2 \) eV for \( n = 2; \ 9E_1 = 38.7 \) eV for \( n = 3; \) and \( 16E_1 = 68.8 \) eV for \( n = 4. \)
(b) The wave functions and energy levels will be like those shown in Figure 6.3.

22. Lacking an explicit equation for finite square well energies, we will approximate using the infinite square well formula. In order to contain three energy levels the depth of the
More precise analysis gives $E = \frac{\hbar^2}{2mL^2} \approx 45.4$ MeV.

24. Using the same notation as the text, from the boundary condition $\psi(x = 0) = \psi(x = 0)$ we have $Ae^0 = Ce^0 + De^0$ or $A = C + D$. From the condition $\psi'(x = 0) = \psi'(x = 0)$ we have $\alpha A = ikC - ikD$. Solving this last expression for $A$ and combining with the first boundary condition gives $C + D = \frac{ik}{\alpha}C - \frac{ik}{\alpha}D$ or after rearranging $\frac{C}{D} = \frac{ik + \alpha}{ik - \alpha}$.

28. We must normalize by evaluating the triple integral of $\psi^* \psi : \iiint \psi^* \psi \, dx \, dy \, dz = 1$ with $\psi(x, y, z)$ given by Equation (6.47) in the text. We can evaluate the iterated triple integral

$$A^2 \int_0^L \sin^2 \left( \frac{\pi x}{L} \right) dx \int_0^L \sin^2 \left( \frac{\pi y}{L} \right) dy \int_0^L \sin^2 \left( \frac{\pi z}{L} \right) dz = A^2 \left( \frac{L}{2} \right)^3 = 1.$$  

Solving for $A$ we find $A = \left( \frac{2}{L} \right)^{3/2}$.

30. One possible solution is $n_1 = 2$, $n_2 = 1$, $n_3 = 1$. Letting either of the other two quantum numbers be equal to 2 (with the remaining two equal to 1) gives the same result for $A$. For the choice given here, the wave function becomes $\psi = A \sin \left( \frac{2\pi x}{L} \right) \sin \left( \frac{\pi y}{L} \right) \sin \left( \frac{\pi z}{L} \right)$. To normalize, integrate $\psi^* \psi = \psi^2$ over the entire box $A^2 \int_0^L \sin^2 \left( \frac{2\pi x}{L} \right) dx \int_0^L \sin^2 \left( \frac{\pi y}{L} \right) dy \int_0^L \sin^2 \left( \frac{\pi z}{L} \right) dz = A^2 \left( \frac{L}{2} \right)^3 = 1$ Solving for $A$, we find $A = \left( \frac{2}{L} \right)^{3/2}$. It is interesting that this is the same result as for the ground state (Problem 28).