1. A laser beam exerts a force of $2 \times 10^{-11} \text{N}$ on a completely absorbing surface. What is the power of the laser?
   
   a) $6.67 \times 10^{-20} \text{ W}$
   b) $1.5 \times 10^{19} \text{ W}$
   c) $2 \times 10^{-11} \text{ W}$
   d) $2 \times 10^8 \text{ W}$
   e) $6 \times 10^{-3} \text{ W}$

   \[ P = \frac{\Delta P}{\Delta t} = \frac{n E_{ph}}{c} = \frac{1}{c} \left( \frac{n E_{ph}}{\Delta t} \right) \]

2. A $^{336}\text{U}$ nucleus undergoes fission and breaks up into two middle-mass fragments, $^{140}\text{Xe}$ and $^{96}\text{Sr}$. By what percentage does the total nuclear surface area change during this process?

   a) 15%  b) 36%  c) 25%  d) zero  e) 40%

   \[ r = r_0 A^{1/2} \]
   \[ S = 4\pi r^2 \times A^{2/3} \]

3. An observer is in a closed laboratory in the gravity-free environment of outer space. She wishes to determine whether the laboratory is at rest or in motion at a constant velocity.
   
   a) She can find out by performing the Michelson-Morley experiment.
   b) She can find out by bouncing a perfectly elastic ball off the wall in various directions and measuring the velocity before and after the bounce.
   c) None of the other answers will work.
   d) She can find out by comparing two different clocks in the laboratory over a period of time.
   e) She can find out by measuring her mass.

4. Relative on the earth, rocket A moves to the right at 0.50$c$, and rocket B moves to the left at 0.75$c$. What is the speed of rocket B as measured by observers in rocket A?

   a) 0.91$c$  b) 1.25$c$  c) 0.25$c$  d) 0.40$c$  e) 0.18$c$

   \[ V = \frac{0.5 c + 0.75 c}{1 + 0.5 \times 0.75} \]

5. The vacuum wavelength of photon A is twice that of photon B. What is the ratio of the speed of photon A to that of photon B?

   a) $1/4$  b) $1/2$  c) 4  d) 2  e) 1

   \[ c \text{ is a constant.} \]
1 year = 3.15 x 10^7 (sec).

6. A radioactive source consists of 10^{22} atoms. It is observed that 10^{11} atoms decay per second. What is the half-life of the radioactive material?

- a) 2200 years
- b) 1740 years
- c) 3200 years
- d) 1520 years
- e) 10^{-11} years

\[ \lambda = \frac{1}{t} \left| \frac{dN}{dt} \right| = \frac{10^{11}}{10^{22}} = 10^{-11} \text{ (sec}^{-1} \text{)} \]

\[ t_{1/2} = \frac{\ln 2}{\lambda} = 6.93 \times 10^6 \text{ (sec)} = 2197 \text{ (years)} \]

7. A proton with 1 GeV total energy has a de Broglie wavelength of:

- a) 3.6 \times 10^{-15} m
- b) 9.0 \times 10^{-16} m
- c) 4.1 \times 10^{-24} m
- d) 1.2 \times 10^{-15} m
- e) 2.4 \times 10^{-16} m

\[ \lambda = \frac{h}{p} = \frac{h}{pc} = \frac{1.24 \times 10^{-6} \text{ eV}}{0.3458 \times 10^9 \text{ eV}} = 3.57 \times 10^{-15} \text{ (m)} \]

8. After 2 hours, \( \frac{1}{16} \) of the initial amount of a certain radioactive isotope has decayed. The half-life of the isotope is

- a) 15 min.
- b) 30 min.
- c) 45 min.
- d) 60 min.
- e) 2 hours

\[ \frac{1}{16} = \left( \frac{1}{2} \right)^2 \left( \frac{t}{t_{1/2}} \right) \]

\[ t_{1/2} = \frac{1}{4} t = 0.5 \text{ (hour)} \]

9. At \( T = 300 \text{ K} \) \( (k_B T = 0.0259 \text{ eV}) \), the band gap of Si is 1.14 eV, and the Fermi energy is in the middle of its band gap as shown. What is the probability that an energy level at the bottom of the conduction band is occupied?

- a) 2.8 x 10^{-10}
- b) 3.6 x 10^9
- c) 0.242
- d) 0.361
- e) 0.046

\[ f(E_c) = \frac{1}{e^{\frac{(E_c - E_F)}{k_B T}} + 1} \]

\[ n = \frac{1}{e^{\frac{E_g}{k_B T}} + 1} \]

\[ = e^{-\frac{1.14}{0.0259 \times 10^2}} \]

\[ = e^{-22} \]

\[ \propto 2.7 \times 10^{-10} \]
10. The energy levels for an electron in an unknown potential are as shown. If five electrons are placed in the potential so as to minimize their energy, the total energy of the 5 electrons is:

\[ 2\times (1+3) + 5 \]

- a) 5 eV
- b) 29 eV
- c) 43 eV
- d) 11 eV
- e) 13 eV

\[ \approx 13 \text{ eV} \]

Assume the electrons do not interact with each other.

11. The diagram shows five possible absorption transitions between the \( \nu=0 \) and \( \nu=1 \) vibrational levels of a diatomic molecule. Which transition is forbidden?

- a) I
- b) II
- c) III
- d) IV
- e) V

\[ \Delta l = \pm 1 \]

\[ l : l = 1 \to l = 1 \]

\[ \Delta l = 0 \]

12. A 20 keV photon scatters off a free electron through an angle of 135°. The wavelength of the scattered photon is about:

\[ \lambda = \frac{h}{E} = \frac{1240 \text{ nm} \cdot \text{eV}}{2 \times 10^4 \text{ eV}} \approx 62 \text{ pm} \]

\[ \Rightarrow \lambda' = 62 + 4 = 66 \text{ (pm)} \]

13. When a nonrelativistic beam of electrons is focused down to a diameter \( \Delta x \), it is found that the electrons' velocities perpendicular to the beam can be measured to a precision of \( \Delta v_x \). If a nonrelativistic beam of protons is focused to the same diameter, the protons' perpendicular velocities can be determined to a precision of about \( \Delta v_p \approx \)

- a) \( \Delta v_x \)
- b) \( \frac{1}{2} \Delta v_x \)
- c) 2000\( \Delta v_x \)
- d) \( \Delta \frac{v_x}{2000} \)
- e) 10\( \Delta v_x \)

\[ \Delta p = m \Delta v \]

\[ \Rightarrow \Delta p = \frac{h}{\lambda} \]

\[ \Rightarrow \Delta v_p = \frac{m_e \Delta v_e}{m_p} \approx \frac{1}{\tau} \Delta v_e \]
14. For a metal at $T = 0 \text{ K}$, the probability that a state 0.50 eV below the Fermi level is occupied is:

- a) 0
- b) $5.0 \times 10^{-9}$
- c) $5.0 \times 10^{-6}$
- d) $5.0 \times 10^{-3}$
- e) 1

15. Two events occur 1000 m apart with an intervening time interval of 0.60 μs. The speed of a reference frame in which they occur at the same time is:

- a) 0
- b) 0.18 c
- c) 0.36 c
- d) 1.8 c
- e) None of the other answers.

16. Muons have a half life of $1.52 \times 10^{-6}$ sec as measured by an observer at rest with respect to them. Muons from cosmic rays traveling at 0.98 c toward earth. On average an observer on top of a mountain with 2 km elevation records 1000 muons per second. How many muons per second will the observer at sea level record?

- a) 45
- b) 54
- c) 360
- d) 540
- e) 940

17. Calculate the ratio of the vibrational to rotational energies, $E_{\text{vib}}/E_{\text{rot}}$, for a diatomic molecule in the state $(n = 0, \ell = 1)$, where $n$ and $\ell$ are the vibrational and rotational quantum numbers, respectively. The rotational constant of the molecule is $\hbar^2/(2I) = 6.3 \times 10^{-4}$ eV, and its vibrational resonance energy is $\hbar \omega = 0.1$ eV.

- a) 0
- b) $2.5 \times 10^{20}$
- c) 160
- d) 40
- e) 80

$$E_{\text{rot}} = \frac{\hbar^2}{2I} (\ell + 1) \ell$$

$$E_{\text{vib}} = \hbar \omega (n + \frac{1}{2})$$

$$\Rightarrow \left( \frac{E_{\text{vib}}}{E_{\text{rot}}} \right) = \frac{\frac{\hbar^2}{2I} \cdot 2}{\hbar \omega \cdot \frac{1}{2}} = \frac{4 \times 6.3 \times 10^{-4}}{0.1} = 0.252$$

$$\Rightarrow \left( \frac{E_{\text{vib}}}{E_{\text{rot}}} \right) \approx 40$$
18. Which of the following elements would you consider as a dopant for Silicon (valence structure $3s^23p^2$) if you wanted to make a p-type semiconductor?
   a) Phosphorus (valence structure $3s^23p^3$)
   b) Arsenic (valence structure $4s^24p^3$)
   c) Helium (valence structure $1s^2$)
   d) Aluminum (valence structure $3s^23p^1$)
   e) Carbon (valence structure $2s^22p^2$)

19. Consider the nuclear reaction $^{28}_{14}\text{Si} + n \rightarrow ^{14}_{7}\text{N} + ^{4}_{1}\text{H}$. What is the threshold kinetic energy of the neutron for this reaction to occur (supposing that $^{28}_{14}\text{Si}$ is at rest)? You may use atomic masses $M(\text{Si}) = 26.9867048$ u, $M(\text{N}) = 14.0030740$ u.
   a) $K_{th} = 10.0$ MeV
   b) $K_{th} = 10.4$ MeV
   c) $K_{th} = 5.2$ MeV
   d) $K_{th} = 20.8$ MeV
   e) This reaction cannot occur.

20. Only one of the following reactions or decays CANNOT occur. Which is it? (Assume that the projectile always has enough energy for the reaction to occur, if other applicable conservation laws permit).
   a) $\nu_e + p \rightarrow n + e^+ \times$
   b) $\tau^+ \rightarrow e^+ + \nu_e + \bar{\nu}_e \checkmark$
   c) $\mu^- \rightarrow e^- + \nu_\mu + \bar{\nu}_e \checkmark$
   d) $p + p \rightarrow p + p + \pi^0 \checkmark$
   e) $\pi^- \rightarrow \mu^- + \bar{\nu}_\mu \checkmark$

************* End of multiple-choice problems *************
21. Escape speed from near Earth's surface is $1.1 \times 10^4$ m/s.
   (a) Find the temperature required for helium gas to have an rms speed equal to that escape speed.
   (b) Your answer to (a) is much higher than temperatures on Earth. Why then does helium escape from Earth's atmosphere?
   (c) The escape speed of a particle near the Sun's surface is $6.2 \times 10^5$ m/s. Most of the gas there is atomic hydrogen. Find the rms speed of a hydrogen atom, assuming the Sun's surface temperature is 5800 K. Compare your answer with the escape speed.

   \[
   \text{(a) } \nu_{\text{rms}} = \sqrt{\frac{3k_B}{m}} \Rightarrow T = \frac{m
   \nu_{\text{rms}}^2}{3k_B} = 1.94 \times 10^4 \text{ K.}
   \]

   \[
   \text{(b) for RT (book) } \nu_{\text{rms}}^2 = 1.3 \times 10^3 \text{ (m/s)} < \nu_{\text{esc}}.
   
   \nu_{\text{rms}} \text{ is not much smaller than } \nu_{\text{esc}} \text{ so the probability of escaping is small but finite, hence}
   \]

   \[
   \text{(c) } \nu_{\text{rms}} = \sqrt{\frac{3k_B}{m}} = \sqrt{\frac{0.15 \text{ eV}}{1.38 \times 10^{-23} \text{ J/K}}} = 1.2 \times 10^5 \text{ (m/s)}.
   
   \text{Comparable with } \nu_{\text{esc}} \text{ on Sun.}
   \]

22. The transition from the $\ell = 2$ to the $\ell = 1$ state in CO is accompanied by the emission of a $9.55 \times 10^{-4}$ eV photon.
   (a) Use this information to find the rotational inertia of the CO molecule.
   (b) What is the bond length between the C and O atoms?

   \[
   \text{(a) } I_{\text{rot}} = \frac{h^2}{2I} \ell (\ell + 1)
   \]

   \[
   \Delta E = E^{(2)}_{\text{rot}} - E^{(1)}_{\text{rot}} = \frac{h^2}{2I} \left[ 6 - 2 \right] = \frac{2h^2}{I}
   \]

   \[
   \Rightarrow I = \frac{2h^2}{\Delta E} = \frac{2 \times (9.55 \times 10^{-4})^2}{8.55 \times 10^{-4} \times 1.6 \times 10^{-19}} = 1.45 \times 10^{-46} \text{ (kg\cdotm^2)}
   \]

   \[
   \text{(b) } \mu = \frac{m_c + m_o}{m_c + m_o} \Rightarrow \mu = \frac{12u + 16u}{12 + 16} = 6.857u.
   \]
23. For alpha decay $^{4}_2P \rightarrow ^{4}_2D + \alpha$, here $\alpha$ is Helium 4 nucleus $^4_2$He. The disintegration energy is define as $Q = (M_P - M_D - M_\alpha)c^2$. The kinetic energy of the $\alpha$ particle is $K_\alpha$.
   (a) Use non-relativistic approximation to show that:
   $$K_\alpha = Q \frac{M_D}{M_D + M_\alpha}$$

   (b) Use relativistic formula to show that:
   $$K_\alpha = Q \frac{(M_P + M_D - M_\alpha)}{2M_P}$$

   \[ \text{[a]} \text{ momentum conservation: } M_D v_D = M_\alpha v_\alpha \Rightarrow v_D = \frac{m_\alpha}{m_D} v_\alpha. \]

   \[ \Rightarrow [K_\alpha = Q \frac{M_D}{M_D + M_\alpha}]. \]

   \[ \text{[b]} \text{ pressure conservation. } P_\beta = P_\alpha \quad (\text{we omit } c=1) \]

   \[ \text{[c]} \text{ energy conservation. } M_P = \sqrt{P_\beta^2 + m_\beta^2} = \sqrt{P_\alpha^2 + m_\alpha^2} \Rightarrow M_P^2 - 2M_P \sqrt{P_\alpha^2 + m_\alpha^2} = P_\alpha^2 - m_\alpha^2 \Rightarrow E_\alpha = \frac{1}{2} P_\alpha^2 + m_\alpha^2 \Rightarrow K_\alpha = E_\alpha - m_\alpha = \frac{m_\alpha^2 + m_\beta^2 - m_D^2}{2m_\beta} \]

24. Considering an ideal monoatomic gas in 2 dimension (2D). Use kinetic theory and equipartition theorem to derive the ideal gas law in 2D:

$$P^{2D} A = Nk_B T$$

where $N$ is the number of molecules, $T$ is temperature of the gas, $A$ is the area filled by the gas. Note that in 2 dimension, the "pressure" is defined as "force per unit length", i.e. $P^{2D} = F/L$, and the "volume" is reduced to "area" ($A$).

Consider 1 molecule in 2D chamber ($L \times W$).

$$\langle f^2 \rangle = \frac{2m \langle v^2 \rangle}{L} = \frac{m \langle v^2 \rangle}{L}$$

$$\Rightarrow \text{Pressure } \rho^{2D} = \frac{F}{W} = \frac{m \langle v^2 \rangle}{L \times W} = \frac{m \langle v^2 \rangle}{A}$$

$$\text{Equipartition theorem: } \langle f^2 \rangle = \frac{1}{2} k_B T \Rightarrow \rho^{2D} A = k_B T$$

For $N$ molecules, $P = \frac{N}{2} \rho^{2D} = Nk_B T/A \Rightarrow \rho^{2D} A = Nk_B T$. 