Name ________________ Signature ________________

1. The exam will last from 3:20pm to 4:40pm. Use a #2 pencil to make entries on the answer sheet. Enter the following id information now, before the exam starts.

2. In the section labeled NAME, enter your last name, your first name, and finally your middle initial.

3. Under STUDENT # enter your 9-digit Student ID Number. Under COURSE enter 273. Under CODE enter the exam code given above.

4. During the exam, you may use pencils, a calculator, and ONE 8½ × 11″ sheet of paper with formulas and notes.

5. There are 14 problems on this exam. 12 of them are multiple-choice questions (5 point each). For each multiple-choice question, mark only one answer on the answer sheet. There is no deduction of points for an incorrect answer. So even if you cannot work out the answer to a question, you should make an educated guess.

6. There are 2 open-end problems (40 points total) on this exam. Please write down the solution of each problem on the provided papers. Please write down your name on each extra exam paper.

7. At the end of the exam, hand in only the answer sheet and the exam papers of open-ended problems. Retain this question paper for future reference and study.

8. Useful numerical constants are given on the next page. Before starting the exam, make sure that your copy contains the page of constants and all 16 questions. Bring your exam to the proctor if this is not the case.
Useful information

Elementary charge $e = 1.6 \times 10^{-19} \text{C}$

1 electron volt (eV) = $1.6 \times 10^{-19} \text{J}$

Speed of light $c = 3 \times 10^8 \text{m/s}$

Planck’s constant $h = 6.63 \times 10^{-34} \text{J} \cdot \text{s}$ = 1240 nm·eV/c

$h = \frac{h}{2\pi} = 1.054 \times 10^{-34} \text{J} \cdot \text{s}$

Compton wavelength of electron $\frac{h}{mc} = 0.00243 \text{nm}$

Ground-state energy of hydrogen $= -13.6 \text{eV}$

Rydberg constant of Hydrogen atom $R_H = 0.0109678 \text{nm}^{-1}$

Avogadro’s number $N_A = 6.02 \times 10^{23} \text{molecules/mole}$

Electron mass $= 9.11 \times 10^{-31} \text{kg} = 0.511 \text{MeV/c}^2$

Proton mass $= 1.673 \times 10^{-27} \text{kg} = 938.3 \text{MeV/c}^2$

Neutron mass $= 1.675 \times 10^{-27} \text{kg} = 939.6 \text{MeV/c}^2$

Atomic mass unit 1 $u = 931.5 \text{MeV/c}^2$

Bohr magneton $\mu_B = \frac{e \hbar}{2m_e} = 9.274 \times 10^{-24} \text{J/T} = 5.788 \times 10^{-5} \text{eV/T}$

Vacuum permittivity $\varepsilon_0 = 8.854 \times 10^{-12} \text{F/m}$

Powers of ten:

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**Multiple-choice problems**

1. The Pauli exclusion principle states:
   a) No two electrons in an atom can have the same principal quantum number.
   b) No two electrons can have all the same quantum numbers.
   c) Identical particles are indistinguishable.
   d) Electrons are Fermions.
   e) The wavelength associated with a particle is inversely proportional to its momentum.

2. An infinite one-dimensional square well contains five (5) noninteracting electrons (which are Fermions) in their lowest possible energy states. If the kinetic energy of the first level in the well is 5 eV, what is the total kinetic energy of the five-electron system?
   a) 25 eV
   b) 125 eV
   c) 10 eV
   d) 95 eV
   e) 30 eV

3. Two electrons with opposite spin are in an $L = 0$ orbital. A magnetic field of 1 T is present. What is the energy separation between the two electron levels?
   a) $3.2 \times 10^{-2}$ eV
   b) $2.9 \times 10^{-5}$ eV
   c) $1.16 \times 10^{-4}$ eV
   d) $5.65 \times 10^{-3}$ eV
   e) $1.54 \times 10^{-8}$ eV

4. What is the correct ground-state electron configuration of Mg ($Z = 12$)?
   a) $1s^22s^22p^63s^2$
   b) $1s^22p^62d^4$
   c) $1s^22s^22p^62d^2$
   d) $1s^22s^23s^23p^6$
   e) $1s^22s^23s^24s^22p^4$
5. The minimum energy required to ionize the $K$ shell ($n = 1$) of plutonium ($Z = 94$) is about:
   a) 120 keV  
   b) 1.28 keV  
   c) 132 eV  
   d) 0.14 eV  
   e) 115 MeV

6. In an atom with two electrons, one electron has orbital angular momentum quantum number $\ell = 2$ and the other has $\ell = 3$. Which of the following is NOT a possible value of the total angular momentum quantum number $j$ (neglect spin) ?
   a) 4  
   b) 3  
   c) 2  
   d) 1  
   e) 0

7. The ground state energy of an electron in a one-dimensional infinite square well with zero potential energy in the interior and infinite potential energy at the walls is 2.0 eV. If the width of the well is doubled, the ground state energy will be:
   a) 0.5 eV  
   b) 1.0 eV  
   c) 2.0 eV  
   d) 4.0 eV  
   e) 8.0 eV

8. Calculate the ground state energy for an electron in a one-dimensional box with a width of 0.05 nm.
   a) 10 eV  
   b) 75 eV  
   c) 24 eV  
   d) 150 eV  
   e) 54 eV
9. A particle is confined to a finite one-dimensional square well as shown. Which of the wave functions shown is a possible stationary state of this potential?

\[ V(x) = \begin{cases} 0 & 0 \leq x < a \\ V_0 & x = a \end{cases} \]

- \( a) \)
- \( b) \)
- \( c) \)
- \( d) \)
- \( e) \)

10. Two electrons in lithium (Z = 3) have as their quantum numbers \((n, \ell, m_\ell, m_s)\) the values \((1, 0, 0, \pm \frac{1}{2})\). The third electron is in its lowest possible energy state, i.e. its ground state. Which of the following are possible quantum numbers for this third electron? (There is no magnetic field present.)

- \( a) \) \((2, 0, 0, \frac{1}{2})\)
- \( b) \) \((2, 1, 1, \frac{1}{2})\)
- \( c) \) \((2, 1, 1, -\frac{1}{2})\)
- \( d) \) \((1, 1, 1, -\frac{1}{2})\)
- \( e) \) \((3, 0, 0, \frac{1}{2})\)
11. The statements below all agree with the Bohr model for the hydrogen atom. One of them disagrees with the Schrödinger model for the hydrogen atom. Which is it?

a) In the hydrogen atom, the relationship between total energy, \( E \), potential energy, \( U \), and kinetic energy, \( K \), is given by \( E = K + U \).

b) The frequency, \( f \), of a photon emitted when an electron makes a transition from the \( i^{th} \) orbit to the \( j^{th} \) orbit is given by \( hf = E_i - E_j \).

c) The orbital angular momentum of the lowest possible energy level, i.e. the ground state, is \( L = \hbar \).

d) The potential energy function for the atom is given by \( V(r) = -\frac{k e^2}{r} \).

e) The energy for the ground state of hydrogen is \(-13.6 \) eV.

12. A particle with energy \( E \) is confined by finite potential energy walls to a one-dimensional trap from \( x = 0 \) to \( x = L \). Its wave function in the region \( x > L \) has the form: (Here \( A \) is a constant)

\[
\psi(x) = \begin{cases} 
A \sin(kx) & \text{if } x \leq L \\
A & \text{if } x > L
\end{cases}
\]

a) \( \psi(x) = A \sin(kx) \)

b) \( \psi(x) = A e^{kx} \)

c) \( \psi(x) = A e^{-kx} \)

d) \( \psi(x) = A e^{ikx} \)

e) \( \psi(x) = A \)
13. How does an external magnetic field affect the 2P state in Hydrogen?

a) It splits the state into 4 \( j = 3/2 \) and 2 \( j = 1/2 \) states. The energy spacing between adjacent \((j = 1/2, m_j)\) states and adjacent \((j = 3/2, m_j)\) are equal.

b) It splits the state into 4 \( j = 3/2 \) and 2 \( j = 1/2 \) states. The energy spacing between adjacent \((j = 1/2, m_j)\) states are larger than those between adjacent \((j = 3/2, m_j)\).

c) It splits the state into 4 \( j = 3/2 \) and 2 \( j = 1/2 \) states. The energy spacing between adjacent \((j = 1/2, m_j)\) states are smaller than those between adjacent \((j = 3/2, m_j)\).

d) It splits the state into 6 \( j = 5/2 \) and 4 \( j = 3/2 \) states. The energy spacing between adjacent \((j = 5/2, m_j)\) states are larger than those between adjacent \((j = 3/2, m_j)\).

e) It does not affect the state.

*************** End of multiple-choice problems ***************
Please use additional exam papers to answer the open-ended problems.

14. The wave function for the first excited state $\psi_1$ for the simple harmonic oscillator is $\psi_1 = Ax e^{-ax^2/2}$. Normalize the wave function to find the value of the constant $A$. Determine $\langle x \rangle$, $\langle x^2 \rangle$ and $\Delta x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2}$.

15. Find out the classical probability distribution function $P(x)$ of simple harmonic oscillator with spring constant $k$, mass $m$. Assume the oscillation amplitude is $a$. The probability $P(x)dx$ of finding the particle is proportional to the time $dt$ that the particle spend on each position $x$ within interval $dx$. Here $dt$ depends on the speed $v(x)$ at the location.