Physics 273' – Honor Physics IIIa
Final Exam
Friday, December 23, 2016
Prof. Weida Wu

Name ___________________ Signature __________________

1. The exam will last from 12:00pm to 3:00pm. Use a # 2 pencil to make entries on the answer sheet. Enter the following id information now, before the exam starts.

2. In the section labeled NAME, enter your last name, your first name, and finally your middle initial.

3. Under STUDENT # enter your 9-digit Student ID Number. Under COURSE enter 273. Under CODE enter the exam code given above.

4. During the exam, you may use pencils, a calculator, and THREE 8½” × 11” sheet of paper with formulas and notes.

5. There are 24 problems on this exam. 20 of them are multiple-choice questions (3 points each). For each multiple-choice question, mark only one answer on the answer sheet. There is no deduction of points for an incorrect answer. So even if you cannot work out the answer to a question, you should make an educated guess.

6. There are 4 open-end problems (10 points each) on this exam. Please write down your name on each extra exam paper.

7. At the end of the exam, hand in the answer sheet and the exam papers.

8. Useful numerical constants and formula are given on the next few pages. Before starting the exam, make sure that your copy contains the page of constants and all 24 questions. Bring your exam to the proctor if this is not the case.

**Useful formula:**

\[
\int_0^{+\infty} e^{-t^2} \, dt = \frac{\sqrt{\pi}}{2}, \quad \int_0^{+\infty} t e^{-t^2} \, dt = \int_0^{+\infty} t^3 e^{-t^2} \, dt = \frac{1}{2}, \quad \int_0^{+\infty} t^2 e^{-t^2} \, dt = \frac{\sqrt{\pi}}{4}.
\]

\[
\int_0^1 x^2 e^{-2x} \, dx = 0.0808
\]
Elementary charge $e = 1.6 \times 10^{-19}$ C
1 electron volt (eV) = $1.6 \times 10^{-19}$ J
Speed of light $c = 3 \times 10^8$ m/s
Planck's constant $\hbar = 6.63 \times 10^{-34}$ J·s, $hc = 1240 \text{ nm·eV}$
$h = \frac{\hbar}{2\pi} = 1.054 \times 10^{-34}$ J·s = $6.582 \times 10^{-16}$ eV·s
Compton wavelength of electron $\frac{\hbar}{m_e c} = 0.00243 \text{ nm} = 2.43 \text{ pm}$

Ground-state energy of hydrogen atom = $-13.6 \text{ eV}$
Rydberg constant of Hydrogen atom $R_H = 0.0109678 \text{ nm}^{-1}$

Avogadro's number $N_A = 6.02 \times 10^{23}$ molecules/mole

Electron mass = $9.11 \times 10^{-31}$ kg = $0.511 \text{ MeV/c}^2$

Bohr magneton $\mu_B = \frac{e\hbar}{2m_e} = 9.274 \times 10^{-24}$ J/T = $5.788 \times 10^{-5}$ eV/T

Atomic mass unit 1 u = $1.66 \times 10^{-27}$ kg = $931.5 \text{ MeV/c}^2$

Proton mass = $1.673 \times 10^{-27}$ kg = $938.3 \text{ MeV/c}^2 = 1.007276$ u

Neutron mass = $1.675 \times 10^{-27}$ kg = $939.6 \text{ MeV/c}^2 = 1.008665$ u

Mass of Hydrogen atom = $1.007825$ u, $\alpha$ ($^4_2\text{He}$) mass = $4.0026032$ u

### Powers of ten:

- Femto (f): $10^{-15}$
- Pico (p): $10^{-12}$
- Nano (n): $10^{-9}$
- Micro (µ): $10^{-6}$
- Milli (m): $10^{-3}$
- Centi (c): $10^{-2}$
- Kilo (k): $10^3$
- Mega (M): $10^6$
- Giga (G): $10^9$
- Tera (T): $10^{12}$

### THE LEPTONS (all spin $\frac{1}{2}$)

<table>
<thead>
<tr>
<th>Mass (MeV)</th>
<th>Common decays</th>
<th>$L_e$</th>
<th>$L_\mu$</th>
<th>$L_\tau$</th>
<th>Antiparticle</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e^-$ (.511)</td>
<td>Stable</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>$e^+$</td>
</tr>
<tr>
<td>$\nu_e$ (0?)</td>
<td>Stable (?)</td>
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<td>0</td>
<td>0</td>
<td>$\bar{\nu}_e$</td>
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<tr>
<td>$\mu^-$ (106)</td>
<td>$e^-\bar{\nu}<em>e\nu</em>\mu$</td>
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<td>1</td>
<td>0</td>
<td>$\mu^+$</td>
</tr>
<tr>
<td>$\nu_\mu$ (0?)</td>
<td>Stable (?)</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>$\bar{\nu}_\mu$</td>
</tr>
<tr>
<td>$\tau^-$ (1777)</td>
<td>$\pi^-\pi^0\nu_\tau$, $e^-\bar{\nu}<em>e\nu</em>\tau$, $\mu^-\bar{\nu}<em>\mu\nu</em>\tau$</td>
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<td>0</td>
<td>1</td>
<td>$\tau^+$</td>
</tr>
<tr>
<td>$\nu_\tau$ (0?)</td>
<td>Stable (?)</td>
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<td>0</td>
<td>1</td>
<td>$\bar{\nu}_\tau$</td>
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</tbody>
</table>

### THE QUARKS (all spin $\frac{1}{2}$)

Baryon number = $+1/3$ for all quarks, and $-1/3$ for all antiquarks
S = strangeness, C = charm, B = bottomness, T = topness

<table>
<thead>
<tr>
<th>Name</th>
<th>Symbol</th>
<th>Charge</th>
<th>S</th>
<th>C</th>
<th>B</th>
<th>T</th>
<th>Antiparticle</th>
</tr>
</thead>
<tbody>
<tr>
<td>Down</td>
<td>$d$</td>
<td>$-1/3$</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>$\bar{d}$</td>
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<tr>
<td>Up</td>
<td>$u$</td>
<td>$+2/3$</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>$\bar{u}$</td>
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<tr>
<td>Strange</td>
<td>$s$</td>
<td>$-1/3$</td>
<td>-1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>$\bar{s}$</td>
</tr>
<tr>
<td>Charm</td>
<td>$c$</td>
<td>$+2/3$</td>
<td>0</td>
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<td>0</td>
<td>0</td>
<td>$\bar{c}$</td>
</tr>
<tr>
<td>Bottom</td>
<td>$b$</td>
<td>$-1/3$</td>
<td>0</td>
<td>0</td>
<td>-1</td>
<td>0</td>
<td>$\bar{b}$</td>
</tr>
<tr>
<td>Top</td>
<td>$t$</td>
<td>$+2/3$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>+1</td>
<td>$\bar{t}$</td>
</tr>
</tbody>
</table>
HADRONS (strongly interacting particles)

Baryon number = +1 for baryons, −1 for antibaryons, 0 for all others
S = strangeness, C = charm, B = bottomness

SOME BARYONS (all are fermions: half-integer spin)

<table>
<thead>
<tr>
<th>Mass (MeV)</th>
<th>Common decays</th>
<th>S</th>
<th>C</th>
<th>B</th>
<th>Antiparticle</th>
</tr>
</thead>
<tbody>
<tr>
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<td>n (940)</td>
<td>$pe^-\nu_e$</td>
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<td>0</td>
<td>0</td>
<td>$\bar{n}$</td>
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<tr>
<td>$\Lambda$ (1116)</td>
<td>$p\pi^-, n\pi^0$</td>
<td>−1</td>
<td>0</td>
<td>0</td>
<td>$\Lambda$</td>
</tr>
<tr>
<td>$\Sigma^+$ (1189)</td>
<td>$p\pi^0, n\pi^+$</td>
<td>−1</td>
<td>0</td>
<td>0</td>
<td>$\Sigma$</td>
</tr>
<tr>
<td>$\Sigma^0$ (1193)</td>
<td>$\Lambda\gamma$</td>
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<td>0</td>
<td>$\Sigma^0$</td>
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<td>$\Sigma^-$ (1197)</td>
<td>$n\pi^-$</td>
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<td>0</td>
<td>$\Sigma^-$</td>
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<tr>
<td>$\Xi^0$ (1315)</td>
<td>$\Lambda\pi^0$</td>
<td>−2</td>
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<tr>
<td>$\Xi^-$ (1321)</td>
<td>$\Lambda\pi^-$</td>
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<td>0</td>
<td>$\Xi^-$</td>
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<tr>
<td>$\Omega^-$ (1672)</td>
<td>$\Lambda K^-, \Xi^0\pi^-$</td>
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<td>0</td>
<td>0</td>
<td>$\Omega$</td>
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<td>$\bar{\Lambda}_c$</td>
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<tr>
<td>$\Lambda_b^+$ (5624)</td>
<td>Various</td>
<td>0</td>
<td>0</td>
<td>−1</td>
<td>$\bar{\Lambda}_b$</td>
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</tbody>
</table>

SOME MESONS (all are bosons: integer spin)

<table>
<thead>
<tr>
<th>Mass (MeV)</th>
<th>Common decays</th>
<th>S</th>
<th>C</th>
<th>B</th>
<th>Antiparticle</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi^+$ (140)</td>
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<td>0</td>
<td>$\pi^-$</td>
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<tr>
<td>$\pi^0$ (135)</td>
<td>$\gamma\gamma$</td>
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<td>0</td>
<td>0</td>
<td>Self</td>
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<tr>
<td>$\eta^0$ (547)</td>
<td>$2\gamma, 3\pi^0,\ldots$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>Self</td>
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<tr>
<td>$K^+$ (494)</td>
<td>$\mu^+\nu_\mu, \pi^+\pi^0$</td>
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<td>0</td>
<td>$K^-$</td>
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<tr>
<td>$K^0$ (498)</td>
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<td>0</td>
<td>$K^0$</td>
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<tr>
<td>$D^+$ (1869)</td>
<td>$K^\pm + \ldots, K^0 + \ldots$</td>
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<td>+1</td>
<td>0</td>
<td>$D^-$</td>
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<tr>
<td>$D^0$ (1865)</td>
<td>$K^\pm + \ldots, K^0 + \ldots$</td>
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<td>+1</td>
<td>0</td>
<td>$D^0$</td>
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<tr>
<td>$D_s^+$ (1969)</td>
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<td>+1</td>
<td>0</td>
<td>$D_s^-$</td>
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<td>$J/\psi$ (3097)</td>
<td>Various</td>
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<td>0</td>
<td>Self</td>
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<td>$B^+$ (5279)</td>
<td>$D^\pm + \ldots, D^0 + \ldots$</td>
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<td>0</td>
<td>+1</td>
<td>$B^-$</td>
</tr>
<tr>
<td>$B^0$ (5279)</td>
<td>$D^\pm + \ldots, D^0 + \ldots$</td>
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<td>+1</td>
<td>$B^0$</td>
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<tr>
<td>$\Upsilon$ (9460)</td>
<td>Various</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>Self</td>
</tr>
</tbody>
</table>
*************** Multiple-choice problems ******************

1. A nearby star is 4 light years away from the earth. If an astronaut travels from the earth to the star with a speed of 0.8 c, how much time (unit: years) will have elapsed on his clock at the end of the journey?
   a) 2.4  b) 5  c) 4  d) 3  e) 8.3

2. A Michelson interferometer operates with a laser of wavelength $\lambda = 600$ nm. One arm of the interferometer has a mirror at the end of a copper bar, and the light path goes out parallel to the bar and is reflected by the mirror back along the same path. As the bar is cooled from $T = 300$ K to $T = 77$ K, a total of 125 dark fringes are seen to pass through the eyepiece. It is concluded that the copper bar has contracted by:
   a) $7.5 \times 10^{-5}$ m
   b) $3.75 \times 10^{-5}$ m
   c) $1.5 \times 10^{-4}$ m
   d) $3 \times 10^{-4}$ m
   e) $1.9 \times 10^{-5}$ m

3. Two events occur 100 m apart with an intervening time interval of 0.60 $\mu$s. The speed of a reference frame in which they occur at the same coordinate is:
   a) 0
   b) 0.25c
   c) 0.56c
   d) 1.1c
   e) 1.8c

4. A double-slit experiment of electron uses a slit spacing $d$ and electrons with energy $E$. Here $E \ll m_e$, i.e. non-relativistic limit. On a screen located a long distance $L$ away ($L \gg d$), the interference pattern shows that adjacent bright fringes are separated by a small distance $y$. If the electron energy $E$ is then doubled, what will be the new separation between adjacent bright fringes?
   a) $4y$
   b) $y/\sqrt{2}$
   c) $y/4$
   d) $y/2$
   e) $\sqrt{2}y$
5. The threshold wavelength for photoemission in calcium is 384 nm. If the light of wavelength 200 nm is used, what will be the photoelectric stopping potential $V_s$ in volts?
   a) $V_s < 2.1$
   b) $2.1 \leq V_s < 3.1$
   c) $3.1 \leq V_s < 4.1$
   d) $4.1 \leq V_s < 5.1$
   e) $V_s \geq 5.1$

6. In a metal, at the absolute zero of temperature
   a) all motion ceases
   b) the Fermi energy is zero
   c) the Fermi speed is zero
   d) the average kinetic energy of the conduction electrons is zero
   e) the average kinetic energy of the conduction electrons differs significantly from zero

7. The figure shows the Fermi function for a solid at two different temperatures $T_A$ and $T_B$. From the shape of these curves we can tell that:
   a) $T_A < T_B$
   b) $T_A > T_B$
   c) $T_B = 0 \text{ K}$
   d) states above the Fermi energy are more likely to be occupied at $T_A$ than at $T_B$.
   e) the solid is an insulator.

8. A 1.00 g sample of pure KCl from the chemistry stockroom is found to be radioactive and to decay at an absolute rate $R$ of 1600 counts/s. The decay is traced to the element potassium and in particular to the isotope $^{40}\text{K}$, which constitutes 1.18% of normal potassium. The molecular weight of KCl is 74.9 g/mole. The disintegration constant $\lambda$ is
   a) $1.12 \times 10^{-17} \text{s}^{-1}$
   b) $1.69 \times 10^{-17} \text{s}^{-1}$
   c) $1.96 \times 10^{-17} \text{s}^{-1}$
   d) $2.30 \times 10^{-17} \text{s}^{-1}$
   e) none of these
9. 50 neutrons with mass $m$ are placed in a one dimensional square well, of size $\ell$. Neglect any interaction between neutrons. At zero temperature, the highest energy neutron will have an energy of: [hint: neutrons are fermions.]

   a) $625(h^2 \pi^2 / 2m\ell^2)$
   b) $(h^2 \pi^2 / 2m\ell^2)$
   c) $50(h^2 \pi^2 / 2m\ell^2)$
   d) $25(h^2 \pi^2 / 2m\ell^2)$
   e) $2500(h^2 \pi^2 / 2m\ell^2)$

10. Neutrinos
   a) are very high energy photons
   b) are thought to exist, but have not yet actually been detected
   c) carry little energy
   d) interact weakly with matter
   e) are anti-neutrons

11. In a doped semiconductor material of the p-type
   a) the density of conduction electrons far exceeds the density of holes
   b) the holes carry negative charge
   c) the current is carried mainly by the electrons
   d) the current is carried mainly by the holes
   e) the valence band is completely full

12. In a hydrogen atom, in the ground state, described by $\psi_1(r) = (\pi r_0^2)^{-\frac{1}{2}} e^{-r/r_0}$, approximately what is the ratio of the probability $P(r > r_0)$ that the electron will be found beyond the Bohr radius, $r_0$, to the probability $P(r \leq r_0)$ that it is found inside the distance?

   a) 1.0  b) 0.5  c) 2.1  d) 1.5  e) 2.5

13. Pions ($\pi^-$) have a half life of $2.2 \times 10^{-8}$ sec as measured by an observer at rest with respect to them. An observer in a laboratory sees a beam of pions traveling at 0.995 c. As the beam passes him he counts 1000 pions per second. How many pions per second will be left after the beam travels 10 m (measured by the observer in the laboratory) further?

   a) 360  b) 510  c) 630  d) 900  e) 860
14. In an atom with two electrons on \( n = 3 \) level, one electron on \( p \) orbital and the other is on \( d \) orbital. Which of the following spectroscopy symbols \( (n^2S + L_J) \) is NOT a possible state?
   a) \( 3^1P_1 \)  
   b) \( 3^3D_3 \)  
   c) \( 3^3F_2 \)  
   d) \( 3^3P_1 \)  
   e) \( 3^1S_0 \)

15. Which of the following is not true?
   a) The rest mass of a hydrogen atom is less than the sum of the rest masses of an electron and a proton.
   b) The rest mass of a helium nucleus is less than the sum of the rest masses of two protons and two neutrons.
   c) The rest mass of an iron nucleus \( (^{56}_{26}\text{Fe}) \) is less than the sum of the rest mass of 13 \( \alpha \) particles and 4 neutrons.
   d) The rest mass of a uranium nucleus \( (^{235}_{92}\text{U}) \) is less than the sum of the rest masses of \( ^{110}_{44}\text{Ru} \), \( ^{122}_{48}\text{Cd} \), and 3 neutrons.
   e) The rest mass of a neutron is more than the sum of the rest masses of a proton and an electron.

16. For a semiconductor with a band gap \( E_g = 0.4 \text{ eV} \) and the Fermi level in the middle of the gap, by how many times does the occupancy of a state at the very bottom of the conduction band increase when temperature is raised from 300 to 600 K?
   a) 47
   b) 0.000435
   c) 0.02
   d) 2
   e) 15.38

17. The ionization energy of hydrogen is 13.6 eV. What would the ionization energy be if the electron were replaced by a muon \( (\mu^-) \) ? Mass of muon is \( m_\mu \approx 105.658 \text{ MeV}/c^2 \), and mass of electron is \( m_e \approx 0.511 \text{ MeV}/c^2 \)
   a) 0.066 eV
   b) 13.6 eV
   c) 2.82 keV
   d) 196 eV
   e) 0.94 eV
18. Which of the following are acceptable descriptions of a state of Hydrogen atom?
I \( n = 2, l = 1, m_l = 1, s = 1, m_s = 0 \)
II \( n = 2, l = 0, m_l = 0, s = 1/2, j = 3/2, m_j = -3/2 \)
III \( n = 2, l = 1, s = 1/2, j = 3/2, m_j = -3/2 \)
IV \( n = 2, l = 1, m_l = -1, s = 1/2, m_s = -1/2 \)
   a) All
   b) I and III
   c) II and III
   d) III and IV
   e) II and IV

19. Only one of the following reactions or decays can occur. Which one? In case of reactions, sufficient kinetic energy is available in the initial state.
   a) \( \pi^- + p \rightarrow n + \bar{n} \)
   b) \( p + p \rightarrow p + n + \pi^0 \)
   c) \( p + p \rightarrow \pi^+ + \pi^+ \)
   d) \( p + p \rightarrow \pi^+ + \pi^- \)
   e) \( e^- \rightarrow \tau^- + \nu_e + \bar{\nu}_\tau \)

20. The energy of the \( n = 2 \) level of hydrogen is 10 eV above the \( n = 1 \) ground state. At 11,600 K (\( k_B T = 1 \) eV) what fraction of hydrogen atoms are in an \( n = 2 \) state? [Note: think carefully here]
   a) \( 1 \times e^{-10} \)
   b) \( 2 \times e^{-10} \)
   c) \( 3 \times e^{-10} \)
   d) \( 4 \times e^{-10} \)
   e) \( 1 \times e^{-5} \)

*************** End of multiple-choice problems ***************
open-ended problems

21. The bond length of $N_2$ molecules is approximately 0.11 nm. The effective spring constant of the N-N covalent bond is approximately $k = 2247 \text{ N/m}$. Mass of nitrogen atom is $m_N = 14 \text{ u}$.

(5 points) (a) Use Maxwell-Boltzmann factors to find the population ratio $P_1/P_0$ of the $\ell = 0$ and $\ell = 1$ states of rotational motion of $N_2$ molecules at room temperature (300 K). Moment of inertia of diatomic molecules is $I = \mu R^2$, where $\mu$ is the reduced mass and $R$ is the bond length.

(5 points) (b) Use Maxwell-Boltzmann factors to find the population ratio $P_1/P_0$ of the $n = 0$ and $n = 1$ states of vibrational motion of $N_2$ at room temperature (300 K).
22. One way to decide whether Maxwell-Boltzmann statistics are valid for an ideal gas is to compare the de Broglie wavelength $\lambda$ of a typical molecule with the average inter-molecule spacing $d$. If $\lambda \geq d$ then Maxwell-Boltzmann statistics are not valid.

(3 points) (a) Using RMS speed $v_{\text{rms}}$ to show that:

$$\lambda = \frac{h}{\sqrt{3mk_BT}}$$

Here $m$ is the mass of the molecule and $T$ is the gas temperature.

(3 points) (b) Use the fact that $V/N = d^3$ to show that the equality $\lambda = d$ condition can be expressed as:

$$\frac{N}{V} \frac{h^3}{(3mk_BT)^{3/2}} = 1$$

(4 points) (c) Use above results to determine the critical temperature $T_c$ of helium gas when Maxwell-Boltzmann statistics are not valid. Assume the gas density is fixed at the standard condition value (i.e. 22.4 liters for 1 mole of gas).
Low energy nuclear reaction kinematics, i.e. kinetic energies are typically much lower than the rest energies. Consider the reaction \( x + X \rightarrow y + Y \), where \( X \) is at rest and \( x \) is the projectile with speed \( v_x \). Define energy release \( Q \) as: \( Q = (M_x + M_X - M_y - M_Y)c^2 \). For exothermic \((Q < 0)\) reaction, show that the minimum kinetic energy \( K_{th} = \frac{1}{2}m_xv_x^2 \) needed to initiate the reaction is:

\[
K_{th} = -Q \left( \frac{M_x + M_X}{M_X} \right)
\]
24. Considering a monoatomic ideal gas in 2 dimension (2D).

**(5 points)** (a) Use Boltzmann factor to derive the velocity distribution function \( P(v_x, v_y) \) of the 2D ideal gas with proper normalization, i.e.

\[
\int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} P(v_x, v_y) dv_x dv_y = 1
\]

[hint: start from 1D velocity distribution.]

**(5 points)** (b) Use the result of (a) to derive the speed distribution function \( P(v) \) (where \( v = \sqrt{v_x^2 + v_y^2} \)) of the 2D ideal gas.