Physics 273’ – Honor Physics IIIa
Second Hourly Exam
Wednesday, November 16, 2016
Prof. Weida Wu

Name _________________ Signature ________________

1. The exam will last from 3:20pm to 4:40pm. Use a #2 pencil to make entries on the answer sheet. Enter the following id information now, before the exam starts.

2. In the section labeled NAME, enter your last name, your first name, and finally your middle initial.

3. Under STUDENT # enter your 9-digit Student ID Number. Under COURSE enter 273. Under CODE enter the exam code given above.

4. During the exam, you may use pencils, a calculator, and ONE $8\frac{1}{2}'' \times 11''$ sheet of paper with formulas and notes.

5. There are 14 problems on this exam. 12 of them are multiple-choice questions (5 point each). For each multiple-choice question, mark only one answer on the answer sheet. There is no deduction of points for an incorrect answer. So even if you cannot work out the answer to a question, you should make an educated guess.

6. There are 2 open-end problems (40 points total) on this exam. Please write down the solution of each problem on the provided papers. Please write down your name on each extra exam paper.

7. At the end of the exam, hand in only the answer sheet and the exam papers of open-ended problems. Retain this question paper for future reference and study.

8. Useful numerical constants are given on the next page. Before starting the exam, make sure that your copy contains the page of constants and all 14 questions. Bring your exam to the proctor if this is not the case.
Useful information

Elementary charge \( e = 1.6 \times 10^{-19} \text{C} \)

1 electron volt (eV) = 1.6 x 10^{-19} J

Speed of light \( c = 3 \times 10^8 \text{m/s} \)

Planck’s constant \( h = 6.63 \times 10^{-34} \text{J} \cdot \text{s} = 1240 \text{nm} \cdot \text{eV}/c \)

\( h = h/2\pi = 1.054 \times 10^{-34} \text{J} \cdot \text{s} \)

Compton wavelength of electron \( \frac{h}{mc} = 0.00243 \text{nm} \)

Ground-state energy of hydrogen = −13.6 eV

Rydberg constant of Hydrogen atom \( R_H = 0.0109678 \text{nm}^{-1} \)

Avogadro’s number \( N_A = 6.02 \times 10^{23} \text{molecules/mole} \)

Electron mass = 9.11 x 10^{-31} kg = 0.511 MeV/c^2

Bohr magneton \( \mu_B = \frac{e\hbar}{2mc} = 9.274 \times 10^{-24} \text{J/T} = 5.788 \times 10^{-5} \text{eV/T} \)

Proton mass = 1.673 x 10^{-27} kg = 938.3 MeV/c^2

Neutron mass = 1.675 x 10^{-27} kg = 939.6 MeV/c^2

Atomic mass unit \( 1 \text{u} = 931.5 \text{MeV/c}^2 \)

Vacuum permittivity \( \varepsilon_0 = 8.854 \times 10^{-12} \text{F} \cdot \text{m}^{-1} \)

Powers of ten:

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1. An electron and a positron can form a bound state analogous to the hydrogen atom. Such an $e^+e^-$ bound state is called positronium. What is the binding energy of positronium?
   
   a) 13.6 eV  
   b) 27.2 eV  
   c) 3.4 eV  
   d) 9.8 eV  
   e) 6.8 eV

2. A particle has a normalized wave function $\psi(x) = (2x/L^2)^{1/2}$ in the region $0 < x < L$. What is the probability that at any instant the particle lies between $x = 0$ and $x = L/3$?
   
   a) About 67%  
   b) About 82%  
   c) About 33%  
   d) About 11%  
   e) Zero

3. A particle is in the $n^{th}$ energy level of an infinite square well. The probability density for finding the particle at the center of the well is zero if:
   
   a) the particle is in the ground state.  
   b) $n$ is odd.  
   c) $n$ is odd and the spin of the particle is $\frac{1}{2}$.  
   d) $n$ is even.  
   e) The probability density is never zero.

4. In the ground state, the four valence electrons in atomic $^{28}_{14}$Si are best represented by:
   
   a) 3s:  
   b) 3s:  
   c) 3s:  
   d) 3s:  
   e) 3s:  
   
   3p:  
   3p:  
   3p:  
   3p:  
   3p:
5. Compared to the $n^{th}$ level of an **infinite** square well, the $n^{th}$ level of the **finite** square well:
   a) has more kinetic energy.
   b) has more nodes.
   c) has a smaller spatial extent.
   d) has a less kinetic energy.
   e) has a greater curvature.

6. A hydrogen atom is in the 3d state ($n = 3$, $\ell = 2$). How many distinct values of the $z$ component of the electron’s total angular momentum are possible? (Include electron spin.)
   a) 1  
   b) 2  
   c) 4  
   d) 6  
   e) 8

7. Which of the following levels of the hydrogen atom is not split into two components by spin-orbit coupling?
   a) 2s  
   b) 2p  
   c) 4d  
   d) 4p  
   e) 6f

8. In the ground state of the hydrogen atom, what is the probability that the electron will be found within a sphere whose radius is that of the first Bohr orbit? [remember that for the ground state $\psi(r) = \frac{l}{\sqrt{\pi r_B}} e^{-r/r_B}$ and that $\int_0^2 x^2e^{-x}dx = 2(1 - 5/e^2)$]
   a) 16%  
   b) 20%  
   c) 28%  
   d) 32%  
   e) 64%
9. The diagram indicates several possible transitions among energy levels in a hydrogen atom involving the emission or absorption of a photon. Which transition(s) are NOT allowed?

a) III only  
b) II and V  
c) I only  
d) IV only  
e) I and II

10. Which of the following is a correct wavefunction for the potential well shown? The total energy of the particle is indicated by the dashed line.
11. Which diagram best represents the wave function of a particle in the \( n = 3 \) level of a FINITE one-dimensional square well?

![Diagrams](image)

a)  

b)  

c)  

d)  

e)  

12. A complex atom is in a \( 4^3F_4 \) state. An external magnetic field (B) is turned on. What will be the spacing between adjacent magnetic substates?

a)  \( 0.67\mu_B B \)

b)  \( 1.25\mu_B B \)

c)  \( 1.50\mu_B B \)

d)  \( 2.00\mu_B B \)

e)  \( 1.33\mu_B B \)

************** End of multiple-choice problems **************
Please use additional exam papers to answer the open-ended problems.

13. The Morse potential is a good approximation for a real potential to describe diatomic molecules. It is given by $V(r) = D(1 - e^{-\alpha(r-x_e)})^2$ where $D$ is the molecular dissociation energy, and $r_e$ is the equilibrium distance between the atoms. For small vibrations, $r - r_e$ is small, and $V(r)$ can be expanded in a Taylor series to reduce to a simple harmonic potential.

(10 points) (a) Find the lowest term of $V(r)$ in this expansion and show that it is quadratic in $(r - r_e)$. [Hint: define $\delta \equiv r - r_e$ to simplify the notation. Here $\delta \ll 1$. Recall the Taylor expansion of exponential function $e^x \approx 1 + x + x^2/2 + ...$]

(5 points) (b) Find the expression characteristic angular frequency $\omega$ in terms of $D$ and $\alpha$, and write down the expression of eigen energy within the harmonic oscillation approximation.

14. Consider the semi-infinite-well potential in which $V = \infty$ for $x \leq 0$, $V = 0$ for $0 < x < L$, and $V = V_0$ for $x \geq L$.

(5 points) (a) Show that possible wave functions are $\psi_I = A\sin(kx)$ inside the well and $\psi_{II} = Be^{-\kappa x}$ for $x > L$, where $k = \sqrt{2mE}/\hbar$ and $\kappa = \sqrt{2m(V_0 - E)}/\hbar$.

(5 points) (b) Show that the application of the boundary conditions gives

$$\kappa = -k \cot(kL)$$

(10 points) (c) Define $\alpha \equiv \kappa L$, $\beta \equiv kL$, and $\alpha_0 \equiv \sqrt{2mV_0}/\hbar \cdot L$. Derive the master equation for solving $\beta$ graphically. (hint: show that $\alpha^2 + \beta^2 = \alpha_0^2$)

(10 points) (d) Sketch the functions on both sides of the master equation in (c). Discuss the condition of only one bound state inside the semi-infinite-well.