Lecture 21

Solid State Physics

Nov. 26, 2007

Electrical Conduction in Solids

$$m\,\frac{\mathrm{d}v}{\mathrm{d}t} = eE - \frac{m}{\tau}\,v$$

in steady state: $$\frac{\mathrm{d}v}{\mathrm{d}t} = 0 \quad \Rightarrow \quad \frac{m}{\tau}\,v_d = eE \quad \Rightarrow \quad v_d = \frac{e\tau}{m}\,E$$

current density: $$j \;=\; e n_e v_d \;=\; \frac{e^2 n_e \tau}{m}\,E \;=\; \frac{1}{\rho}\,E$$

resistivity: $$\rho \;=\; \frac{m}{e^2 n_e \tau}$$

Ohm's Law: $$E \;=\; \rho j \qquad LE \;=\; \frac{L\rho}{A}\,I \quad \Rightarrow \quad V = RI$$

Mean Free Path

Let atom have a cross sectional area, σ

fraction of area covered by atoms in a thickness $\mathrm{d}x$: $\quad \sigma n_a \mathrm{d}x$

Let $N(x)$ be number of electrons that have not scattered after a distance x

$$\mathrm{d}N(x) = -\sigma n_a \mathrm{d}x \quad \Rightarrow \quad N(x) = N_0 e^{-x\sigma n_a} = N_0 e^{-x/l}$$

mean free path: $$l = \frac{1}{\sigma n_a} \qquad \tau = \frac{l}{v_{\text{av}}} = \frac{1}{\sigma n_a v_{\text{av}}}$$

resistivity: $$\rho = \frac{m}{e^2 n_e \tau} = \frac{m \sigma n_a v_{\text{av}}}{e^2 n_e}$$

$$= \frac{m \sigma v_{\text{av}}}{e^2}$$ if one conduction electron per atom

Classical Calculation

$$\rho = \frac{m \sigma v_{\mathsf{av}}}{e^2}$$

$$\sigma \approx \pi a_0^2 \approx 10^{-20}\ \mathrm{m}^2 \qquad \text{where } a_0 \text{ is the Bohr radius}$$

$$v_{\mathsf{av}} = \text{ average thermal velocity} = \sqrt{\frac{8kT}{\pi m}} \approx 10^5\ \mathrm{m/s}$$

This is wrong. It gives a value of resistivity that is about an order of magnitude larger than that of typical metals. More importantly it predicts that the resistivity is proportional to $\sqrt{T}$. Experimental measurement give a resistivity proportional to T.

We need to look at quantum mechanical effects.

Quantum Effects

Quantum mechanics leads to two important differences.

1) The electrons in a metal constitute a Fermi gas. The conduction electrons are in the highest energy levels and, therefore, have energies close to the Fermi energy. The velocity of the conduction electrons is not thermal but is approximately the Fermi velocity

$$v_F = \sqrt{\frac{2E_F}{m}} \approx 10^6 \text{ m/s} \qquad \text{independent of } T.$$

2) The mean free path for scattering in a perfect crystal is infinite. $\Rightarrow$ $\sigma = 0$. Imperfections (impurities, dislocations) give a value of $\sigma \neq 0$ that is independent of T. A larger contribution to σ comes from thermal vibrations of the crystal lattice.

$$\sigma = \pi R_0^2 \qquad M\omega^2 R_0^2 = kT \quad \Rightarrow \quad \sigma = \frac{\pi kT}{M\omega^2}$$

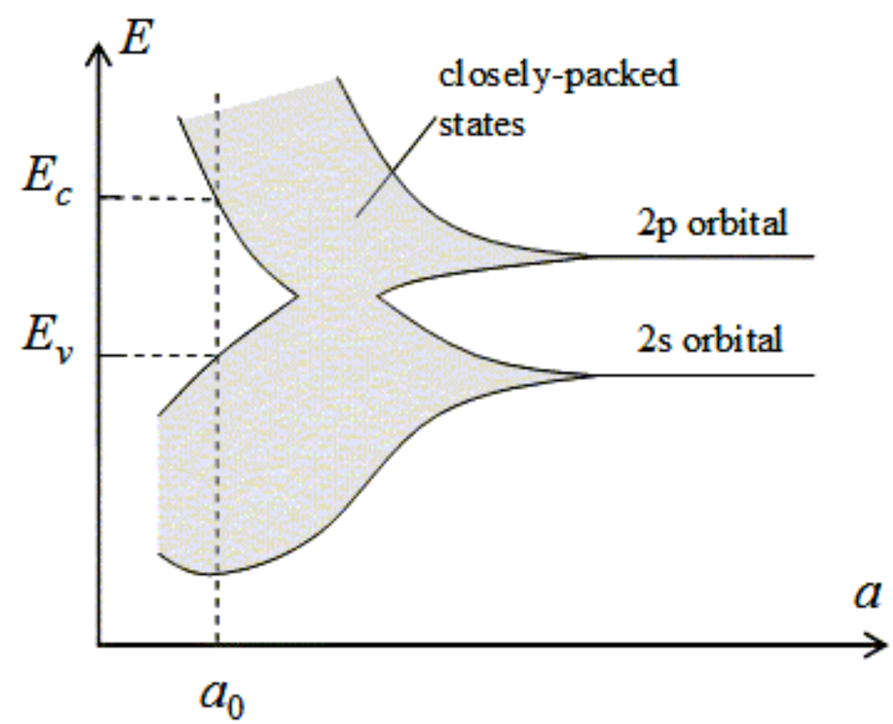
$$\sigma \approx 10^{-22} \text{ m}^2 \qquad \text{at } T = 300 \text{ K}$$

Energy Bands

When atoms are separated by large distances each atom has the same energy levels. Te energy levels are discreet and N-fold degenerate.

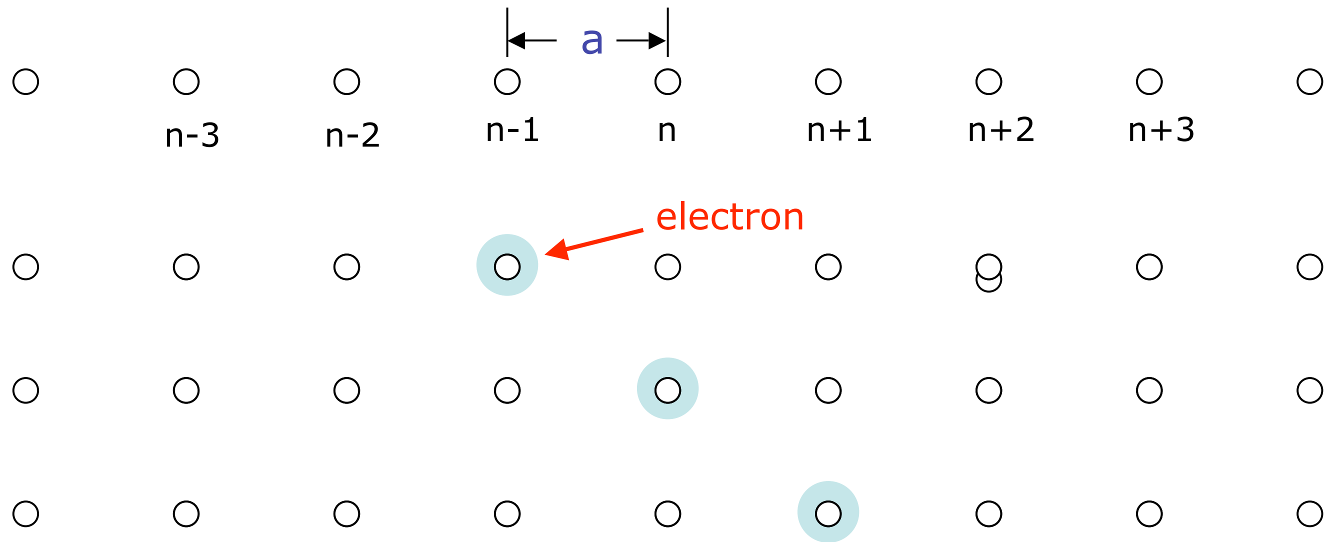
They merge into energy bands as the atoms are brought closer together with "forbidden" energy gaps in between.

This occurs when the wavelength of the electrons becomes comparable to the lattice spacing.



Band Energy Calculation

Consider a one-dimensional lattice of atom



Let $C_n(t)$ be the amplitude for an electron to be at atom n

Let $iA/\hbar$ be the amplitude per unit time for an electron to shift to a neighboring atom.

$$i\hbar \frac{\mathrm{d}C_n(t)}{\mathrm{d}t} = E_0 C_n(t) - AC_{n+1}(t) - AC_{n-1}(t)$$

if $A = 0$ $\qquad C_n(t) = e^{-iE_0 t/\hbar}$

electron just sits at atom with energy E_0

Energy Solutions

Find definite energy solutions when $A \neq 0$

$$C_n(t) \;=\; u(x_n)e^{-iEt/\hbar}$$

$$Eu(x_n) \;=\; E_0 u(x_n) - Au(x_n + a) - Au(x_n - a)$$

Try for a solution: $u(x_n) \;=\; e^{ikx_n}$

$$Ee^{ikx_n} \;=\; E_0 e^{ikx_n} - Ae^{ik(x_n+a)} - Ae^{ik(x_n-a)}$$

$$E \;=\; E_0 - A\left(e^{ika} + e^{-ika}\right) \;=\; E_0 - 2A\cos ka$$

Effective Mass

$$E = E_0 - 2A\cos ka$$

For small ka: $$\cos ka \approx 1 - \frac{k^2a^2}{2}$$

$$E \approx E_0 - 2A + Ak^2a^2$$

$E_0 - 2A$ is a constant energy. We can subtract it and just consider the energy above that of the bottom of the band.

$$E^* = Ak^2a^2 = \frac{Aa^2p^2}{\hbar^2} = \frac{p^2}{2m^*}$$

$$m^* = \frac{\hbar^2}{2Aa^2}$$

typically 2 to 20 times m_e.