

Lecture 3

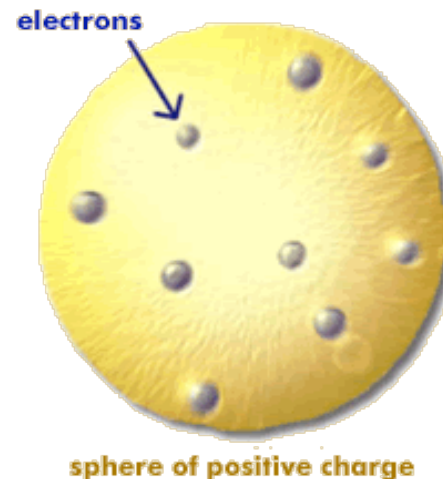
Atoms and the Bohr Model

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Atomic Structure

- In 1898, Thomson discovered the electron.
- It was soon realized that atoms contained electrons. (Electrons could be knocked out of atoms, e.g., the photoelectric effect.)
- An electron is much less massive than an atom so most of mass of the atom is associated with the positive charge.
- Atoms are electrically neutral so they must contain equal amounts of positive and negative charge.
- How could these charges be arranged in a stable configuration?
- First idea was Thomson's "plum pudding" model.

- positive charge uniformly distributed like "pudding"
- electrons distributed like raisins in the "pudding"

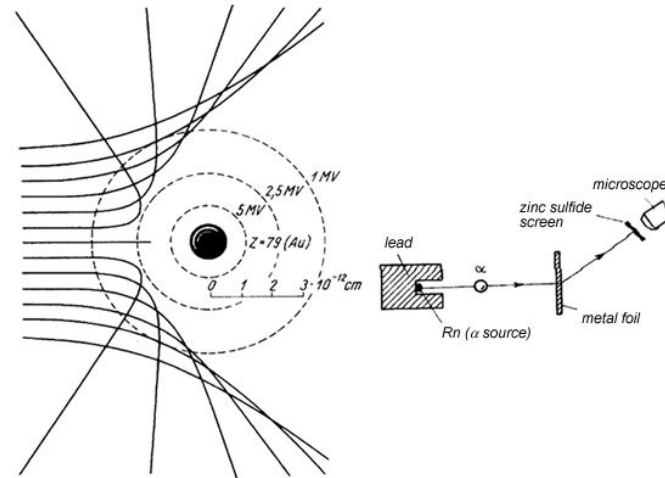


Rutherford Scattering

In 1911, Rutherford scattered α particles, He nuclei, off of atoms.

$$\Delta p = F \Delta t \approx \frac{(2e)(Ze)}{4\pi\epsilon_0 R^2} \frac{2R}{v}$$

$$\theta \approx \frac{\Delta p}{p} \approx \frac{Ze^2}{\pi\epsilon_0 M_\alpha v^2 R}$$



If positive charge distributed over atom, expect only small, 10^{-3} radian, deflections. Note that because $m_e \ll M_\alpha$, the electrons make essentially no contribution to the scattering.

Rutherford found a significant number of scatterings with deflection angles greater than 90° !

⇒ planetary model of atomic structure

light electrons orbiting a very massive positively charged central nucleus.

Problems with Classical Atom

The classical atomic model with electrons orbiting the atomic nucleus like planets orbiting the Sun has two fatal flaws.

1. It cannot account for the discrete spectrum of radiation emitted by excited atoms (see the following slide).
2. The atom would rapidly collapse.
 - An orbiting electron undergoes acceleration (centripetal acceleration).
 - An accelerated charged particle loses energy by emitting electromagnetic radiation.
 - The electron would radiate energy and, in about 10^{-9} s, would spiral into the nucleus.

Atomic Spectra

Excited atoms emit radiation of discrete wavelengths.

For the hydrogen atom, the following empirical formula, the Rydberg-Ritz formula, was found in 1885.

$$\frac{1}{\lambda} = \text{Ry} \left(\frac{1}{n^2} - \frac{1}{m^2} \right)$$

where $\text{Ry} = 1.10 \times 10^7 \text{ m}^{-1}$ is the Rydberg constant and n and m are integers.

Of course, these also correspond to discrete frequencies and energies.

$$\frac{f}{c} = \text{Ry} \left(\frac{1}{n^2} - \frac{1}{m^2} \right) \quad \frac{E}{ch} = \text{Ry} \left(\frac{1}{n^2} - \frac{1}{m^2} \right)$$

Bohr Model

Taking the planetary model of the atom Bohr made the assumption that the angular momentum of the electron about the nucleus is quantized.

$$L = n\hbar$$

In the correct theory of quantum mechanics, the idea of electron orbits is completely wrong. (Remember that in quantum mechanics an object does not have a trajectory.)

Also, although in quantum mechanics angular momentum is quantized, Bohr's formula above is wrong.

Nevertheless, the Bohr model was an important historical step on the way toward developing the correct theory of quantum mechanics and, essentially by dumb luck, it gives several correct numerical results. It's worth spending a lecture on before we go on to do quantum mechanics correctly.

Just remember.

The Bohr model is wrong.

Quantization of Orbital Velocity

Electron bound to nucleus by electrostatic force.

$$F = \frac{e^2}{4\pi\epsilon_0 r^2}$$

For orbiting electron this equals the centripetal force.

$$\frac{mv^2}{r} = \frac{e^2}{4\pi\epsilon_0 r^2}$$

Angular momentum: $L = mvr = \frac{e^2}{4\pi\epsilon_0 v}$

Quantize angular momentum: $n\hbar = \frac{e^2}{4\pi\epsilon_0 v}$

$$v_n = \frac{e^2}{4\pi\epsilon_0 n\hbar}$$

Fine Structure Constant

A dimensionless constant that determines the strength of electric interactions.

$$\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c} = \frac{1}{137}$$

$$v_n = \frac{\alpha c}{n} = \frac{c}{137n}$$

$$v_n \ll c$$

⇒ we can use non-relativistic quantum mechanics to describe the hydrogen atom!

Quantization of Orbital Radius

$$mv_n r_n = n\hbar$$

$$r_n = \frac{n\hbar}{mv_n} = \frac{n^2\hbar}{\alpha mc}$$

For ground state, $n = 1$:

$$r = \frac{\hbar c}{\alpha mc^2} = \frac{200 \text{ MeV} \cdot \text{fm}}{(1/137)(0.511 \text{ MeV})} = 0.53 \times 10^{-10} \text{ m}$$

Bohr radius:

$$a_0 = 0.53 \times 10^{-10} \text{ m}$$

Quantization of Orbital Energy

$$E = \frac{1}{2}mv^2 - \frac{e^2}{4\pi\epsilon_0 r}$$

$$\frac{e^2}{4\pi\epsilon_0 r^2} = \frac{mv^2}{r} \qquad \frac{e^2}{4\pi\epsilon_0 r} = mv^2$$

$$E = \frac{1}{2}mv^2 - mv^2 = -\frac{1}{2}mv^2$$

$$E_n = -\frac{\alpha^2 mc^2}{2n^2}$$

For ground state, $n = 1$:

$$E = -\frac{0.511 \text{ MeV}}{2(137)^2} = -13.6 \text{ eV}$$

Atomic Spectra

Excited atoms make transitions from a higher energy state, n_2 , to a lower energy state, n_1

$$f_{n_2 \rightarrow n_1} = \frac{E_{n_2} - E_{n_1}}{h} = \frac{\alpha^2 mc^2}{2h} \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

$$\frac{1}{\lambda_{n_2 \rightarrow n_1}} = \frac{\alpha^2 mc^2}{2hc} \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

$$\text{Ry} = \frac{\alpha^2 mc^2}{2hc} = \frac{\alpha^2 mc^2}{4\pi \hbar c}$$

$$= \frac{(0.511 \text{ MeV})}{(4\pi)(137)^2(200 \text{ MeV} \cdot \text{fm})} = 1.1 \times 10^{-8} \text{ fm}^{-1} = 1.1 \times 10^7 \text{ m}^{-1}$$

Agrees with experiment.

The Correspondence Principle

In addition to the Bohr atomic model, Bohr and his colleagues also proposed the [Correspondence Principle](#).

This principle is based on the fact that at the objects at the macroscopic level behave classically. The principle then states that any correct quantum theory must approach the classical theory in the limit of very large n .

In the early days, physicists used this as a guide post in the development of the quantum theory. We'll use it several times in the course as a check.

In the present case of the Bohr atom, we have that for large values of n the spacing between levels becomes very small.

$$E_{n_2} - E_{n_1} = \frac{\alpha^2 mc^2}{2} \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right) \rightarrow 0$$

The levels become essentially indistinguishable and approach the continuum of energies of classical orbits.

Summary of Main Points

- Bohr model early, incorrect, model of atomic structure based on quantization of angular momentum. $L = n\hbar$.

- Bohr model correctly gives:

- non-relativistic velocities: $v_n = \frac{\alpha c}{n} = \frac{c}{137n}$

- size of hydrogen atom: $a_0 = \frac{\hbar c}{\alpha m c^2} = 0.53 \times 10^{-10} \text{ m}$

- ground state of hydrogen: $E = -\frac{1}{2} \alpha^2 m c^2 = -13.6 \text{ eV}$.

- atomic spectra: $h f_{n_2 \rightarrow n_1} = E_{n_2} - E_{n_1} = \frac{\alpha^2 m c^2}{2} \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$

- Correspondence Principle:

Approach classical results when $n \rightarrow \infty$.