Homework Assignment #8
Physics 273

13.1) Show, by finding the time for half an initial sample of radioactive material to decay, that the half-life $\tau_{1/2}$ is related to the mean lifetime τ according to $\tau_{1/2} = (\ln 2)\tau$.

$$P(t) = e^{-t/\tau} \qquad \frac{1}{2} = e^{-\tau_{1/2}/\tau} \qquad 2 = e^{\tau_{1/2}/\tau}$$

$$\ln 2 = \frac{\tau_{1/2}}{\tau} \qquad \tau_{1/2} = \tau \, \ln 2$$

13.4) Using the data in Appendix A, find the lifetime and the decay rate for the neutron. Do the same for tritium. Suppose you are given a mole of tritium. How many atoms will you have a thousand years later?

$$\tau = \frac{\tau_{1/2}}{\ln 2} \qquad \Gamma = \frac{1}{\tau}$$

For neutron: $\tau_{1/2} = 10.4 \text{ min} \quad \Rightarrow \quad \tau = 15 \text{ min} \quad \Rightarrow \quad \Gamma = 1.1 \times 10^{-3} \text{ s}^{-1}$

For tritium: $\tau_{1/2} = 12.33 \text{ y} \quad \Rightarrow \quad \tau = 17.8 \text{ y} \quad \Rightarrow \quad \Gamma = 1.8 \times 10^{-9} \text{ s}^{-1}$

$$N = N_0 e^{-t/\tau} = 6.02 \times 10^{23}\, e^{-1000 \text{ y}/17.8 \text{ y}}$$

$$= 6.02 \times 10^{23}\, e^{-56} = (6.02 \times 10^{23})(4.8 \times 10^{-25}) = 0.3$$

13.7) A nucleus decays into two channels with probabilities 0.62 and 0.38, respectively. Its lifetime is 20 hours. What are the decay rates into each of the two channels?

$$R = \frac{1}{\tau} = \frac{1}{20\ \text{h}} = 1.4 \times 10^{-5}\ \text{s}^{-1}$$

$$R_1 = 0.62R = 8.7 \times 10^{-6}\ \text{s}^{-1} \qquad R_2 = 0.38R = 5.3 \times 10^{-6}\ \text{s}^{-1}$$

13.9) A particle called the π^0 decays into two energetic gamma rays with a mean life of $(8.4 \pm 0.6) \times 10^{-17}$ s. A recent table quotes a rest-mass energy of 134.9764 ± 0.0006 MeV for the particle. Is there any contradiction between this very accurate determination of mass and the limit imposed by the uncertainty principle involving ΔE?

By the Heisenberg Uncertainty Principle

$$\Delta E \Delta t \sim \hbar \quad \Rightarrow \quad \Delta E \sim \frac{\hbar}{\Delta t} = \frac{\hbar c}{\Delta t c}$$

$$= \frac{(2 \times 10^{-13} \text{ MeV}\cdot\text{m})}{(8.4 \times 10^{-17} \text{ s})(3.0 \times 10^{8} \text{ m/s})} = 8 \times 10^{-6} \text{ MeV}$$

This is less than the mass-energy uncertainty of 6×10^{-4} MeV

13.10) Which of the following atomic transitions is allowed according to the selection rules developed in this chapter?

The selection rules are: $\Delta l = \pm 1$ $\Delta S = 0$

$^2S_{1/2}$	$\longrightarrow$	$^2P_{3/2}$	allowed	$\Delta l = 1$	$\Delta S = 0$
$^3D_2$	$\longrightarrow$	$^1P_1$	forbidden	$\Delta l = -1$	$\Delta S = -1$
$^3D_{3/2}$	$\longrightarrow$	$^4S_{3/2}$	forbidden	$\Delta l = -2$	$\Delta S = -1/2$
$^2D_{3/2}$	$\longrightarrow$	$^2D_{5/2}$	forbidden	$\Delta l = 0$	$\Delta S = 0$
$^4D_{1/2}$	$\longrightarrow$	$^4P_{1/2}$	allowed	$\Delta l = -1$	$\Delta S = 0$
$^3P_0$	$\longrightarrow$	$^1S_0$	forbidden	$\Delta l = -1$	$\Delta S = -1$

13.18) The rate for transitions up and down at a frequency that corresponds to two levels differing in energy by 0.25 eV is found to be a certain value R when the atom is placed in a blackbody cavity at a temperature of 640 K. At what temperature(s) would the rate of the transition be exactly 10 times R? Is it necessary to go to a higher temperature to get an increased rate of induced transitions? Why or why not?

$$R_{\text{up}} = N_A \langle n \rangle p = N_A \left(\frac{1}{e^{\Delta E/kT} - 1} \right) p$$

$$N_0 = N_A + N_{A^*} = N_A + N_A e^{-\Delta E/kT} = N_A \left(1 + e^{-\Delta E/kT} \right)$$

$$N_A = \frac{N_0}{1 + e^{-\Delta E/kT}}$$

$$\Rightarrow \quad R_{\text{up}} = N_0 \left(\frac{1}{1 + e^{-\Delta E/kT}} \right) \left(\frac{1}{e^{\Delta E/kT} - 1} \right) p = N_0 p \left(\frac{1}{e^{\Delta E/kT} - e^{-\Delta E/kT}} \right)$$

$$R'_{\text{up}} = 10 R_{\text{up}} \quad \Rightarrow \quad \frac{1}{e^{\Delta E/kT'} - e^{-\Delta E/kT'}} = 10 \frac{1}{e^{\Delta E/kT} - e^{-\Delta E/kT}}$$

$$\frac{\Delta E}{kT} = \frac{(0.25 \text{ eV})(1.6 \times 10^{-19} \text{ J/eV})}{(1.38 \times 10^{-24} \text{ J/K})(640 \text{ K})} = 4.53$$

$$e^{\Delta E/kT} = 93 \qquad e^{-\Delta E/kT} = 1.2 \times 10^{-2} \qquad \text{so ignore the negative exponential}$$

$$e^{\Delta E/kT'} = e^{\Delta E/kT}/10 = 9.3 \quad \Rightarrow \quad \frac{\Delta E}{kT'} = \ln(9.3)$$

$$T' = \frac{\Delta E}{k \ln(9.3)} = \frac{(0.25 \text{ eV})(1.6 \times 10^{-19} \text{ J/eV})}{(1.38 \times 10^{-24} \text{ J/K})(2.2)} = 1300 \text{ K}$$

13.19) The angular spread of a laser beam is ideally diffraction limited; that is, the angular spread is given by $\Delta\theta = 1.22\lambda/D$, where λ is the wavelength of the light and D is the diameter of the aperture of the source –the diameter of the laser rod. What is the size of a spot projected by the beam at a distance of 1 km for a ruby laser (λ = 700 nm) with a diameter 1 cm? If the beam has a flux corresponding to 10^{18} photons emitted per second, what is the energy deposited per square centimeter per second at a target 1 km away?

$$d = D + L\,\Delta\theta = D + L\,\frac{1.22\lambda}{D}$$

$$= 1.0\text{ cm} + (10^5\text{ cm})\,\frac{(1.22)(7\times10^{-7}\text{ m})}{(10^{-2}\text{ m})} = 9.5\text{ cm}$$

$$\text{energy per cm}^2\text{ per s} = \frac{(N_\gamma\text{ per s})hf}{\text{area}} = \frac{(N_\gamma\text{ per s})hc}{\lambda(\text{area})}$$

$$= \frac{(10^{18}\text{ s}^{-1})(6.63\times10^{-34}\text{ J}\cdot\text{s})(3.0\times10^{8}\text{ m/s})}{(7\times10^{-7}\text{ m})\pi(9.5/2\text{ cm})^2} = 4\times10^{-3}\text{ W/cm}^2$$

13.21) A laser beam with $\lambda = 600$ nm is 2 mm in diameter. If the beam delivers 10^{17} photons per second, what is the power of the laser? What is the radiation pressure this beam can exert? If we think of the beam as a target made of photons, what is the number of photons per cubic centimeter in that target?

$$P = \frac{\mathrm{d}E}{\mathrm{d}t} = \frac{\mathrm{d}N_\gamma}{\mathrm{d}t} hf = \frac{\mathrm{d}N_\gamma}{\mathrm{d}t} \frac{hc}{\lambda}$$

$$= (10^{17}\ N_\gamma/\mathrm{s}) \frac{(6.63 \times 10^{-34}\ \mathrm{J{\cdot}s})(3.0 \times 10^{8}\ \mathrm{m/s})}{(6.0 \times 10^{-7}\ \mathrm{m})} = 3.3 \times 10^{-2}\ \mathrm{W}$$

$$\text{pressure} = \text{energy density} = \text{intensity}/c = (\text{power per area})/c$$

$$= \frac{(3.3 \times 10^{-2}\ \mathrm{W})/3}{\pi (10^{-3}\ \mathrm{m})^2 (3.0 \times 10^{8}\ \mathrm{m/s})} = 3.5 \times 10^{-5}\ \mathrm{N/m^2}$$

$$\text{energy density} = \text{pressure} = 3.5 \times 10^{-5}\ \mathrm{J/m^3}$$

$$N_\gamma / m^3 = \frac{(\text{energy density})}{hf} = \frac{(\text{energy density})\lambda}{hc}$$

$$= \frac{(3.5 \times 10^{-5}\ \mathrm{J/m^3})(6.0 \times 10^{-7}\ \mathrm{m})}{(6.63 \times 10^{-34}\ \mathrm{J{\cdot}s})(3.0 \times 10^{8}\ \mathrm{m/s})} = 1.06 \times 10^{14}\ \mathrm{m^{-3}} = 1.06 \times 10^{8}\ \mathrm{cm^{-3}}$$