Homework Assignment #1
Physics 273

4.2 Chemical processes typically involve energies on the order of 1 eV. What then, is the typical wavelength of electromagnetic radiation emitted in the course of chemical reactions?

$$E = hf$$

$$f\lambda = c \quad \Rightarrow \quad f = c/\lambda \quad \Rightarrow \quad E = hc/\lambda \quad \Rightarrow \quad \lambda = hc/E$$

$$\lambda = \frac{(6.63 \times 10^{-34}\ \text{J}\cdot\text{s})(3.00 \times 10^{8}\ \text{m/s})}{1\ \text{eV}} = \frac{2.00 \times 10^{-25}\ \text{J}\cdot\text{m}}{1.6 \times 10^{-19}\ \text{J}}$$

$$= 1.25 \times 10^{-6}\ \text{m}$$

Nuclear processes involve energies of the order of 1 MeV. Where in the spectrum of electromagnetic radiation are the photons that may be emitted in a nuclear reaction?

Since the energy is 10^6 larger the wavelength is 10^6 smaller.

$\Rightarrow \quad \lambda = 1.26 \times 10^{-12}$ m. This is in the gamma ray range.

4.4 The quantity $\hbar c$ occurs frequently in calculations in quantum mechanics. Express this quantity in MeV·fm. (1 fermi = 1 fm is 10^{-15} m.)

$$\hbar c = \frac{hc}{2\pi} = \frac{(6.63 \times 10^{-34}\ \text{J·s})(3.00 \times 10^{8}\ \text{m/s})}{2\pi}$$

$$= 3.17 \times 10^{-26}\ \text{J·m} = \frac{(3.17 \times 10^{-26}\ \text{J·m})(10^{15}\ \text{fm/m})}{(1.6 \times 10^{-19}\ \text{J/eV})}$$

$$= 1.98 \times 10^{8}\ \text{eV·fm} = 198\ \text{MeV·fm}$$

4.6 A brick wall absorbs all of the photons that strike it.How many photons per second would have to be emitted by a laser that fires a beam of cross-sectional area 1 mm^2 perpendicularly at the wall at the wall in order to exert a pressure of 1 atm on the wall? The laser emits light of wavelength 600 nm.

For photons: $E = pc$. Since the wall is black each incident photon is absorbed and its momentum is transfered to the wall. The force on the wall is then: $F = (\mathrm{d}N/\mathrm{d}t)(p)$ where $\mathrm{d}N/\mathrm{d}t$ is the rate at which photons hit the wall. The pressure is then the force per area:

$$P = (\mathrm{d}N/\mathrm{d}t)\,\frac{p}{A}$$

$$p = h/\lambda \quad \Rightarrow \quad P = (\mathrm{d}N/\mathrm{d}t)\,\frac{h}{\lambda A} \quad \Rightarrow \quad \mathrm{d}N/\mathrm{d}t = \frac{P\lambda A}{h}$$

$$\mathrm{d}N/\mathrm{d}t = \frac{(1\ \mathrm{atm})(600\times10^{-9}\ \mathrm{m})(1\times10^{-6}\ \mathrm{m}^2)}{(6.63\times10^{-34}\ \mathrm{J\cdot s})}$$

$$= \frac{(1.01\times10^{5}\ \mathrm{N/m^2})(600\times10^{-9}\ \mathrm{m})(1\times10^{-6}\ \mathrm{m}^2)}{(6.63\times10^{-34}\ \mathrm{J\cdot s})}$$

$$\approx 10^{26}\ \text{per second}$$

$$\text{Power} = (\mathrm{d}N/\mathrm{d}t)\times(\text{photon energy}) = hf\ \times 10^{26}\ \mathrm{s}^{-1} = \frac{hc}{\lambda}\times 10^{26}\ \mathrm{s}^{-1}$$

$$= \frac{(6.63\times10^{-34}\ \mathrm{J{\cdot}s})(3.00\times10^{8}\ \mathrm{m/s})}{600\times10^{-9}\ \mathrm{m}}\times10^{26}\ \mathrm{s}^{-1} = 3.3\times10^{7}\ \mathrm{W}$$

4.8 Light of frequency 8.5×10^{15} Hz falls on a metal surface. If the maximum energy of the resulting electrons is 1.7 eV, what is the work function of the metal?

$$E_e^{\text{max}} = E_\gamma - \phi = hf - \phi$$

$$\Rightarrow \quad \phi = hf - E_e^{\text{max}} = \frac{(6.63 \times 10^{-34}\ \text{J}\cdot\text{s})(8.5 \times 10^{15}\ \text{s}^{-1})}{(1.6 \times 10^{-19}\ \text{J/eV})} - 1.7\ \text{eV}$$

$$= 35.2\ \text{eV} - 1.7\ \text{eV} = 33.5\ \text{eV}$$

4.12 A 210 MeV photons collides with an electron at rest. What is the maximum energy loss of the photon?

Energy loss when maximum when $E - E'$ is maximum.

$$\Rightarrow \; f - f' \text{ is maximum} \Rightarrow \; \frac{1}{f'} - \frac{1}{f} \text{ is maximum.}$$

$$\frac{1}{f'} - \frac{1}{f} = \frac{h}{m_e c^2}(1 - \cos\theta) \qquad \text{maximum when } \; \cos\theta = -1$$

$$\left(\frac{1}{f'} - \frac{1}{f}\right)_{\max} = \frac{2h}{m_e c^2} \quad \Rightarrow \quad \frac{1}{f'} = \frac{1}{f} + \frac{2h}{m_e c^2} \quad \Rightarrow \quad \frac{1}{E'} = \frac{1}{E} + \frac{2}{m_e c^2}$$

$$\Rightarrow \quad E' = \frac{E\, m_e c^2/2}{E + m_e c^2/2} = \frac{(210 \text{ MeV})(0.511 \text{ MeV}/2)}{210 \text{ MeV}} = 0.255 \text{ MeV}$$

$$\Rightarrow \quad E - E' \approx 210 \text{ MeV}$$

The backward scattering photon will lose almost all of its energy.

For a proton target:

$$E' = \frac{E\, m_p c^2/2}{E + m_p c^2/2} = \frac{(210 \text{ MeV})(936 \text{ MeV}/2)}{(210 \text{ MeV} + 936 \text{ MeV}/2)} = 145 \text{ MeV}$$

$$\Rightarrow \quad E - E' \approx 65 \text{ MeV}$$

4.16 The function $u(f,T)$ is the distribution of the blackbody radiation in terms of frequency; $u(f,T)\,\mathrm{d}f$ is the energy density contained in the frequency interval from f to $f+\mathrm{d}f$. Use the relation between frequency and wavelength to find the function $Y(\lambda,T)$ that describes the distribution in wavelength; $Y(\lambda,T)\,\mathrm{d}\lambda$ is the energy density contained in a wavelength interval from λ to $\lambda+\mathrm{d}\lambda$.

$$u(f,T) = \frac{\mathrm{d}U}{\mathrm{d}f} \qquad Y(\lambda,T) = \frac{\mathrm{d}U}{\mathrm{d}\lambda}$$

where U is the energy density.

$$Y(\lambda,T) = \frac{\mathrm{d}U}{\mathrm{d}\lambda} = \frac{\mathrm{d}U}{\mathrm{d}f}\frac{\mathrm{d}f}{\mathrm{d}\lambda} = u(f,T)\,\frac{\mathrm{d}f}{\mathrm{d}\lambda}$$

$$f = \frac{c}{\lambda} \quad \Rightarrow \quad \frac{\mathrm{d}f}{\mathrm{d}\lambda} = c\,\frac{\mathrm{d}(1/\lambda)}{\mathrm{d}\lambda} = -\frac{c}{\lambda^2}$$

Ignore the minus sign. It just means that $\mathrm{d}f$ and $\mathrm{d}\lambda$ have opposite signs. (λ decrease as f increases and vice-versa.)

$$Y(\lambda,T) = u(f,T)\,\frac{c}{\lambda^2} = \frac{8\pi f^2}{c^3}\,\frac{hf}{e^{hf/kT}-1}\,\frac{c}{\lambda^2}$$

$$= \frac{8\pi h f^3}{\lambda^2 c^2}\,\frac{1}{e^{hf/kT}-1} = \frac{8\pi hc}{\lambda^5}\,\frac{1}{e^{hc/\lambda kT}-1}$$

4.23 The night-adapted eye can detect the glow of a burning cigarette at some 500 m. Assuming that the pupil of the eye is 0.6 cm in diameter and that the cigarette tip is a hemisphere 1 cm in diameter glowing as a blackbody at a temperature of 1000 K, estimate the rate at which the eye receives photons from the cigarette.

The spectrum of radiation given off by the cigarette covers a range but is peaked near the red. Let's take red as the typical photon, $f_{\text{red}} \approx 5 \times 10^{14}$ Hz. Also we'll take the frequency range over which the eye is sensitive to be, $\Delta f \approx 2 \times 10^{14}$ Hz.

The intensity (energy per unit area per unit time) given off by a blackbody is: Intensity $= S = Uc$. In the frequency range Δf the energy density is:

$$U = u(f,T)\Delta f = \frac{8\pi h f^3}{c^3}\frac{1}{e^{hf_{\text{peak}}/kT} - 1}\Delta f = \frac{8\pi h f^3}{c^3}\frac{1}{e^{27} - 1}\Delta f$$

$$= \frac{(8\pi)(6.63 \times 10^{-34}\ \text{J}\cdot\text{s})(5 \times 10^{14}\ \text{s}^{-1})^3}{(3.00 \times 10^8\ \text{m/s})^3(5.3 \times 10^{11})}(2 \times 10^{14}\ \text{s}^{-1}) = 3 \times 10^{-11}\ \text{J/m}^3$$

Intensity at the surface of the blackbody, the 1 cm diameter sphere representing the burning cigarette end:

$$S = (3 \times 10^{-11}\ \text{J/m}^3)(3 \times 10^8\ \text{m/s}) \approx 10^{-2}\ \text{J}\cdot\text{m}^{-2}\cdot\text{s}^{-1}$$

Since the intensity falls off as $1/r^2$, the intensity at the observer's eye is:

$$(10^{-2}\ \text{J}\cdot\text{m}^{-2}\cdot\text{s}^{-1})\left(\frac{5.0 \times 10^{-3}\ \text{m}}{500\ \text{m}}\right)^2 = 10^{-12}\ \text{J}\cdot\text{m}^{-2}\cdot\text{s}^{-1}$$

The power going into the eye is then:

$$(10^{-12}\ \text{J}\cdot\text{m}^{-2}\cdot\text{s}^{-1})(\pi)(3.0 \times 10^{-3}\ \text{m})^2 = 3 \times 10^{-17}\ \text{J}\cdot\text{s}^{-1}$$

The rate of photons going into the eye is this number divided by the energy of a photon. $\Rightarrow$ $E_{\text{red}} = hf = (6.63 \times 10^{-34}\ \text{J}\cdot\text{s})(5 \times 10^{14}\ \text{s}^{-1}) \approx 3 \times 10^{-19}$ J

$$\Rightarrow \text{ Rate of photons into eye } = \frac{(3 \times 10^{-17}\ \text{J}\cdot\text{s}^{-1})}{(3 \times 10^{-19}\ \text{J})} \approx 100\ \text{s}^{-1}$$