You will want to read T&R Chapters 12.1–13.2 before working through this problem set.

1) Given a nucleus with $Z$ protons and $A - Z$ neutrons, its mass is necessarily less than $m_p Z + m_n (A - Z)$, where $m_p$ and $m_n$ are the mass of the proton and neutron, respectively. This surprising result guarantees that the nucleus doesn’t immediate disintegrate into its constituents. The energy required to separate the nucleus into free neutrons and protons is called the binding energy, and it is (approximately) described as a function of $A$ and $Z$ by the Bethe–Weizsäcker formula (Eq. 12.20 of T&R):

$$B \left( \frac{A}{Z} \right) = a_V A - a_A A^{2/3} - 0.72 Z (Z - 1) A^{-1/3} - a_S \frac{(N - Z)^2}{A} + \delta,$$

where $a_V$, $a_A$, $a_S$, and $\delta$ represent empirically determined parameters.

(a) For a given nuclei with mass number $A$, what is the atomic number $Z$ that maximizes its stability?

(b) Explain why this condition implies that the nuclei cannot undergo $\beta^\pm$ decay.

(c) Let $a_S = 23.2$ MeV. Does this condition hold true for $A = 6$? $A = 28$? $A = 44$? Use Appendix 8 in T&R to decide.

2) The initial fusion reaction in the sun is the process $p + p \rightarrow ^2_1 H + e^+ + \nu_e$. Let’s assume a very simple model for the potential energy function where the Coulomb repulsion dominates until the two protons are separated by less than 1 fm (below which the nuclear force takes over and the two protons fuse to make a deuteron, $^2_1 H$, and other products).

(a) Letting one proton be at rest, calculate the kinetic energy needed for the other proton to overcome the Coulomb barrier.

(b) Convert this into a temperature by assuming the protons in the sun constitute a monatomic ideal gas,

$$\overline{K} = \frac{3}{2} k T,$$

where $k$ is Boltzmann’s constant.

(c) The core temperature of the Sun is $1.5 \times 10^7$ K. Compare your answer to the previous question to this number. How do you propose to account for this discrepancy?
(3) Equation 13.10 in T&R describes the threshold kinetic energy for the reaction \( x + X \rightarrow y + Y \), where the target \( X \) is at rest in the lab frame. This is a non-relativistic approximation. Derive the fully-relativistic equation for the general reaction \( 2 \rightarrow \text{many particles} \),

\[
K_{\text{Th}} = -Q \cdot \frac{\text{Sum of all masses}}{2 \times \text{target mass}},
\]

where \( K_{\text{Th}} \) is the threshold kinetic energy of the initial incoming particle and the target is at rest in the lab frame.