Oh, and I suppose it was me who said ‘what harm could it be to give the chickens a book on nuclear physics?’
Initial evidence for the nucleus came from **Rutherford scattering** (1911).

Fired alpha particles (two protons + two neutrons) at a gold foil. Found that, every once in a while, a proton would scatter back as if it had hit a hard target!

Rutherford scattering experiments confirmed that the nuclear size was about 1 fm ($10^{-15}$ m).

This is to compare with an **atomic size** of $10^{-10}$ m.

$\text{fm} = \text{“femtometer” or “Fermi”}$
Fun with quantum mechanics:

If the nucleus has a size of 1 fm, what is the **minimum binding energy** required to keep a proton within the nucleus? Use the uncertainty principle!

\[
\Delta p \gtrsim \frac{\hbar}{2\Delta x} = \frac{0.20 \text{ eV} \cdot \mu\text{m}/c}{2 \cdot (1 \text{ fm})} \approx 100 \text{ MeV}/c
\]

\[
K = \frac{p^2}{2m} = \frac{(100 \text{ MeV}/c)^2}{2 \cdot 938 \text{ MeV}/c^2} \approx 5 \text{ MeV}
\]
So what holds the nucleus together?

Consider the gravitational potential energy between two protons separate by the nuclear distance:

\[ |V| = \frac{Gmm}{r} \approx \frac{(6.7 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}) \cdot (1.7 \times 10^{-27} \text{ kg})^2}{10^{-15} \text{ m}} \]

\[ |V| \approx 2 \times 10^{-49} \text{ J} \approx 10^{-36} \text{ MeV} \]

The binding energy from our earlier argument from the uncertainty principle had to be at least 5 MeV!

There is no way gravity can be responsible.
What about the electromagnetic force?

It has the wrong sign: protons have the same charge, so they repel each other.

There must exist a new force that is stronger than the electromagnetic force that overcomes the Coulomb repulsion of protons and binds them together.

For lack of imagination, we call this force the strong nuclear force.
Properties of the strong nuclear force

The force is very strong, attractive, and definitely not $1/r^2$.

The force is very short range. The force drops to zero beyond ~2 fm.
Properties of the strong nuclear force

The strong force is independent of the electric charge. \( F_{pp} = F_{nn} = F_{pn} = F_{np} \). Protons and nucleons are collectively called **nucleons**.

Only nucleons are subject to the strong force. Electrons do not “feel” it.

How do we know that electrons don’t feel the strong force? Well, for one, everything we learned about the hydrogen atom would be wrong. The solution to the hydrogen atom depended on the \( V \sim qq/r \) potential, which is not the same as the strong potential.
**Missing matter**

The mass of the proton is 938.3 MeV/c\(^2\).
The mass of the neutron is 939.6 MeV/c\(^2\).
The mass of the deuteron (a proton–neutron bound state) is 1875.7 MeV/c\(^2\).

But 938.3 + 989.6 = 1877.9 MeV/c\(^2\).
There is a 2.2 MeV/c\(^2\) mass difference!

This mass difference is known as the **nuclear binding energy**. It is what keeps the nucleus stable.
Recall that the ionization energy for the hydrogen atom is -13.6 eV.
The binding energy is how much energy needs to be injected into the nucleus to break it apart into its constituent protons and neutrons.

This function can be described (more or less) by the Bethe-Weizsacker semi-empirical mass formula. What it doesn’t model well are the spikes.
For $A>20$ or so, $B/A$ is roughly 8 GeV for all nuclides. This is because of the short range nature of the nuclear force.

Qualitatively, each nucleon only interacts with its nearest neighbors (within ~2 fm or so) and not with those further away.

For a small nucleus, all nucleons still bind each other together. But for a large nucleus ($A>20$), only nearest neighbors do.
Spin-dependence of the nuclear force

The nuclear force does have a (weak, if complicated) spin dependence.

For example, the deuteron (proton-neutron bound state) only exists in the $S=1$ state.

But the proton and neutron are both spin-$1/2$ particles, so the total spin could have been either $S=0$ or $S=1$. 
Nuclei have magnetic moments, too!

Just replace $m_e$ with $m_p$ to get the **nuclear magneton**:

$$\mu_N = \frac{e\hbar}{2m_p}$$

The nuclear magneton is $5 \times 10^{-4} \mu_B$ (because the mass of the proton is so much more than the mass of the electron).

However, there’s something odd:

$$\mu_{\text{proton}} = +2.79 \mu_N$$
$$\mu_{\text{neutron}} = -1.91 \mu_N$$

The “g factor” here is not 1 (again). This points to the fact that **protons and neutrons are not fundamental particles**.
Nuclear radii have the form

\[ R = r_0 A^{1/3} \text{ where } r_0 \approx 1.2 \text{ fm} \]

This is quite sensible. The **volume** of the nucleus grows linearly with the number of nucleons \( A \). So the **radius** of the nucleus should grow linearly with the cube root of \( A \).

For instance, Uranium-238 has a radius of

\[ R \approx 1.2 \cdot (238)^{1/3} \approx 7.4 \text{ fm} \]

So essentially all nuclei have a radius between 1-7 fm.
What about the density?

\[ R = r_0 A^{1/3} \rightarrow V = \frac{4}{3} \pi r_0^3 A \]

Mass = \( Zm_p + (A - Z)m_n \approx Am_p \)

\[ \rho = \frac{\text{Mass}}{V} = \frac{Am_p}{\frac{4}{3} \pi r_0^3 A} = \frac{3m_p}{4\pi r_0^3} \approx 1.5 \times 10^{17} \frac{\text{kg}}{\text{m}^3} \]

Notice that the “A” term cancelled, so all nuclei roughly have the same density (about 2.4 billion tons per cubic inch).
Alpha decay occurs because while the alpha particles are confined within the nuclear potential, occasionally they can sneak out.

This occurs through a process known as quantum tunneling. Although classically, the alpha particle is not energetic enough to overcome the potential barrier, quantum mechanically, the wavefunction has some small probability to be found outside.
Beta decay: an electron and a neutrino are emitted:

$$\frac{A}{Z} P \rightarrow \frac{A}{Z+1} D + e^- + \bar{\nu}_e$$

Here, a neutron in the parent turned into a proton, and electron, and a neutrino, but only the electron and neutrino are emitted.

The neutrino has no electric charge. It has (almost) zero mass. It (almost) never interacts with anything. How do we know it exists?
Suppose that we don’t know about the neutrino.

By analogy with our discussion of alpha decay, we have:

\[ Q = (m_P - m_D - m_e)c^2 \]

Beta decay will occur whenever \( m_P > m_D \). It is very common, and occurs for both light and heavy nuclei.

What is the kinetic energy of the electron?

\[ K_e = \frac{m_D}{m_e + m_D}Q \approx Q \quad (\text{since } m_e \ll m_D) \]

If there is no neutrino, red is what is expected. Blue was what was observed.
Dear Radioactive Ladies and Gentlemen,

As the bearer of these lines, to whom I graciously ask you to listen, will explain to you in more detail, how because of the "wrong" statistics of the N and Li6 nuclei and the continuous beta spectrum, I have hit upon a desperate remedy to save the "exchange theorem" of statistics and the law of conservation of energy. Namely, the possibility that there could exist in the nuclei electrically neutral particles, that I wish to call neutrons, which have spin 1/2 and obey the exclusion principle and which further differ from light quanta in that they do not travel with the velocity of light. The mass of the neutrons should be of the same order of magnitude as the electron mass and in any event not larger than 0.01 proton masses. The continuous beta spectrum would then become understandable by the assumption that in beta decay a neutron is emitted in addition to the electron such that the sum of the energies of the neutron and the electron is constant...

I agree that my remedy could seem incredible because one should have seen these neutrons much earlier if they really exist. But only the one who dare can win and the difficult situation, due to the continuous structure of the beta spectrum, is lighted by a remark of my honoured predecessor, Mr Debye, who told me recently in Bruxelles: "Oh, It's well better not to think about this at all, like new taxes". From now on, every solution to the issue must be discussed. Thus, dear radioactive people, look and judge.

Unfortunately, I cannot appear in Tubingen personally since I am indispensable here in Zurich because of a ball on the night of 6/7 December. With my best regards to you, and also to Mr Back.

Your humble servant,

W. Pauli
While alpha decay and gamma decay can be understood through existing physical phenomena (namely, quantum tunneling and excited nuclei, respectively), beta decay cannot.

Beta decay requires a separate force for a complete explanation. This force is called the weak force (because it is weak).
Natural Uranium is about 99.3% U-238 and 0.7% U-235.

\[ _1^0n + _{92}^{235}U \rightarrow _{56}^{144}Ba + _{36}^{89}Kr + _1^0n + _1^0n + _1^0n \]

The energy release in the above process is about **200 MeV**. (compares to about a few eV per atom with combustion).

Remember where U is on the binding energy curve?
1956: C.S. Wu studies beta-decay of Cobalt-60 in a magnetic field.

$$^{60}_{27}\text{Co} \rightarrow ^{60}_{28}\text{Ni} + e^- + \bar{\nu}_e$$

The Co-60 nucleus has an intrinsic spin of 5, while Ni-60 has an intrinsic spin of 4. Electrons and neutrinos are both spin-1/2 particles.

The spin of the Cobalt nucleus tends to align with the B-field direction.

Odd result: the electrons were always emitted opposite the magnetic field, even though they should be emitted in either direction equally often.

This was the first evidence for parity violation.
Now since there are **three** neutrons produced per reaction, each neutron might hit another Uranium nucleus, producing yet more neutrons...

\[
\frac{1}{0}n + \frac{235}{92}U \rightarrow \frac{144}{56}Ba + \frac{89}{36}Kr + \frac{1}{0}n + \frac{1}{0}n + \frac{1}{0}n
\]

This is the basis of a chain reaction.

1 neutron in the first stage
3 neutrons in the second stage
9 neutrons in the third stage
27 neutrons in the fourth stage
81 neutrons in the fifth stage
...

Of course, this is idealized. Fission isn’t nearly so efficient...
What could make fission inefficient?

1. Competing reactions, eg:

2. Capture of neutrons by U-238, instead of U-235, followed by non-fission decay

3. Leakage of neutrons that don’t interact

\[ _0^1n + _{92}^{235}U \rightarrow _{54}^{140}Xe + _{38}^{94}Sr + _0^1n + _0^1n \]

Each of these can prevent a **self-sustaining** chain reaction.
While nothing can be done about competing reactions (that’s determined by quantum mechanics), one can deal with the other issues.

Can **slow down** neutrons to avoid U-238 capture. Often use $D_2O$ or graphite as a “moderator” to slow them down.
Of course, a “critical mass” of fissile material is needed for a chain reaction.

If each neutron leads to N more neutrons on average then the following holds

- N<1: subcritical (no chain reaction)
- N=1: critical (nuclear reactor goal)
- N>1: supercritical (this is what’s needed for a weapon)

Reactors use control rods made of Cd-113 to keep N close to 1.

The problems for reactors are not supercritical reactions, but core meltdown (which can lead to a loss of containment) and waste disposal (which needs to be kept securely confined for 1000’s of years.)
Fusion of hydrogen into helium releases energy: mass converted to kinetic energy:

\[
\begin{align*}
\frac{2}{1}H + \frac{2}{1}H & \rightarrow \frac{3}{2}He + \frac{1}{0}n + 3.3 \text{ MeV} \\
\frac{2}{1}H + \frac{3}{1}H & \rightarrow \frac{4}{2}He + \frac{1}{0}n + 17.6 \text{ MeV}
\end{align*}
\]

Advantages over fission as a fuel:

1. absence of radioactive waste
2. plentiful supply of deuterons compared to U-235

Problems:

Fusion is very difficult to induce: must overcome the Coulomb repulsion. Practically, not yet efficient (more energy required to induce than is produced).
Proton-Proton Cycle in the Sun
Net result of pp-chain

Convert four protons into: one He-4, 2e\(^+\), 2γ, 2ν and 26.7 MeV of kinetic energy!

Fission gives ~1 MeV/nucleon
(0.09\% of mass converted to kinetic energy)

Fusion gives ~7 MeV/nucleon
(0.66\% of mass converted to kinetic energy)
Example

Sun’s power output = 4\times10^{26} \text{ W} = 2\times10^{39} \text{ MeV/s}

Question: if 90\% of this energy is coming from the pp cycle, how many protons are fused every second?

Q=26.7 \text{ MeV}, so:

\[
\frac{\text{reactions}}{\text{second}} = \frac{0.90 \cdot 2 \times 10^{39}}{26.7} = 6.7 \times 10^{27}
\]

So the sun is burning 4\times6.7\times10^{27} = 3\times10^{28} protons every second!
Example

Sun releases $1.3 \times 10^{38}$ neutrinos/second

Earth-Sun distance is $1.5 \times 10^{11}$ m

What is the neutrino flux at the Earth’s surface?

\[
\frac{1.3 \times 10^{38}}{4\pi \cdot (1.5 \times 10^{11} \text{ m})^2} = 5 \times 10^{14} \frac{\text{neutrinos}}{\text{m}^2 \cdot \text{s}}
\]
Recall way back when...

The temperature of the sun ~5500 K, that is photons have energy of a few eV.
But in the solar fusion process, the kinetic energy of the photons is ~few MeV.

The sun is very dense, so photons scatter many times before they reach the surface of the sun. It takes ~a million years for a photon produced at the core to reach the surface.

On the other hand, neutrinos don’t interact at all. They only take 8 minutes to reach the surface of the earth!
\[ ^{1}H + ^{7}Li \rightarrow ^{2}H + e^{+} + \nu_{e} \]
\[ ^{7}H + ^{3}H \rightarrow ^{3}He + \gamma \]

\[ 69\% \quad 31\% \]

\[ ^{3}He + ^{3}He \rightarrow ^{4}He + 2^{1}H \quad ^{3}He + ^{3}He \rightarrow ^{4}Be + \gamma \]

(PPI)

\[ ^{7}Be + e^{-} \rightarrow ^{7}Li + \nu_{e} \quad ^{7}Be + ^{1}H \rightarrow ^{8}B + \gamma \]
\[ ^{7}Li + ^{1}H \rightarrow 2^{3}He \quad ^{8}B \rightarrow ^{8}Be + e^{+} + \nu_{e} \]

(PP II)

\[ ^{8}Be \rightarrow 2^{4}He \]

(PP III)

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Graph showing the neutrino flux as a function of neutrino energy in MeV, with different reactions labeled and their corresponding neutrino energies and fluxes.