SOLUTIONS

Physics 273– Exam II
Wednesday, November 2, 2011
Prof. Mohan Kalelkar

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Your name sticker and **exam code**

1. The exam will last from 1:45pm to 2:50pm. Use a # 2 pencil to make entries on the answer sheet. Enter the following id information now, before the exam starts.

2. In the section labelled NAME, enter your last name, then fill in the empty circle for a blank, then enter your first name, another blank, and finally your middle initial.

3. Under STUDENT # enter your 9-digit Student ID Number. Under COURSE enter 273. Under CODE enter the exam code given above.

4. During the exam, you may use pencils, a calculator, and **ONE** $8\frac{1}{2}'' \times 11''$ sheet of paper with formulas and notes.

5. There are 16 multiple-choice questions on the exam. For each question, mark only one answer on the answer sheet. There is no subtraction of points for an incorrect answer, so even if you cannot work out the answer to a question, you should make an educated guess. At the end of the exam, hand in only the answer sheet. Retain this question paper for future reference and study.

6. Useful numerical constants are given on the next page. Before starting the exam, make sure that your copy contains the page of constants and all 16 questions. Bring your exam to the proctor if this is not the case.

Elementary charge $e = 1.6 \times 10^{-19}\ C$
1 electron volt $(eV) = 1.6 \times 10^{-19}\ J$
Speed of light $c = 3 \times 10^{8}\ m/s$
Planck's constant $h = 6.63 \times 10^{-34}\ J \cdot s = 1240\ nm \cdot eV/c$
$\hbar = h/2\pi$
Compton wavelength of electron $h/mc = 0.0024\ nm$
Ground-state energy of hydrogen $= -13.6\ eV$
Rydberg constant $R = 0.0109678\ nm^{-1}$
Avogadro's number $= 6.02 \times 10^{23}$ molecules/mole
Electron mass $= 9.11 \times 10^{-31}\ kg = 0.511\ MeV/c^2$
Proton mass $= 1.673 \times 10^{-27}\ kg = 938.3\ MeV/c^2$
Neutron mass $= 1.675 \times 10^{-27}\ kg = 939.6\ MeV/c^2$
Atomic mass unit $1\ u = 931.5\ MeV/c^2$
Fine structure constant $\alpha = 1/137$
Bohr magneton $\mu_B = 5.79 \times 10^{-5}\ eV/T$
Energy ordering of subshells:
1s 2s 2p 3s 3p 4s 3d 4p 5s 4d 5p 6s 4f 5d 6p 7s 5f 6d 7p

Values of l for subshells:

s	p	d	f	g	h
0	1	2	3	4	5

Powers of ten:

femto(f) 10^{-15}	pico(p) 10^{-12}	nano(n) 10^{-9}	micro(μ) 10^{-6}	milli(m) 10^{-3}
centi(c) 10^{-2}	kilo(k) 10^{+3}	Mega(M) 10^{+6}	Giga(G) 10^{+9}	Tera(T) 10^{+12}

1. An electron has a **total** energy of 0.7 MeV. What is its DeBroglie wavelength?

→ a) About 0.0026 nm
b) About 0.0010 nm
c) About 0.0066 nm
d) About 0.0018 nm
e) About 0.0014 nm

$$p = \frac{1}{c}\sqrt{E^2 - m^2c^4} = \frac{1}{c}\left[(0.7\,MeV)^2 - (.511\,MeV)^2\right]^{1/2} = .478\,\frac{MeV}{c}.$$

$$\lambda = \frac{h}{p} = \frac{1240\ nm \cdot \frac{eV}{c}}{.478\times 10^6\ \frac{eV}{c}} = .0026\,nm.$$

2. A particle has a wave function given by $\psi(x) = Ax^2$ in the region $0 < x < L$, and zero elsewhere. What is the value of A?

a) $1/L^2$
b) $\sqrt{3/L^3}$
c) $\sqrt{2/L}$
→ d) $\sqrt{5/L^5}$
e) None of the other answers

Normalization : $\int_0^L \psi^*\psi\, dx = 1$

$$\Rightarrow 1 = \int_0^L A^2 x^4\, dx = A^2 \frac{L^5}{5}$$

$$\Rightarrow A = \sqrt{5/L^5}.$$

3. A particle has a wave function given by $\psi(x) = \sqrt{\frac{\pi}{2L}}\sin\frac{\pi x}{L}$ in the region $0 < x < L$, and zero elsewhere. What is the probability that the particle lies between $x = 0$ and $x = L/3$?

a) $1/\sqrt{3}$
b) $1/3$
c) $1/9$
d) $2/9$
→ e) $1/4$

$$P = \int_0^{L/3} \psi^*\psi\, dx = \int_0^{L/3} \frac{\pi}{2L} \sin\frac{\pi x}{L}\, dx$$

$$= \frac{\pi}{2L}\cdot\frac{L}{\pi}\left[-\cos\frac{\pi x}{L}\right]_0^{L/3} = \frac{1}{2}\left[-\frac{1}{2} - (-1)\right] = \frac{1}{4}.$$

4. Which of the following statements are true?

- I: A proton confined within a nucleus cannot be perfectly at rest.
- II: If the wave function of a particle is zero at some point, the particle has zero probability of being found at that point.
- III: The position and momentum of a particle cannot be simultaneously measured to perfect accuracy.

Uncertainty Principle.

a) I and III are true; II is false
→ b) All three statements are true
c) I and II are true; III is false
d) III is true; I and II are false
e) II and III are true; I is false

$\lambda = \frac{h}{p}$. Same $\lambda \Rightarrow$ same p.

So $E = \sqrt{p^2c^2 + m^2c^4}$ is greater for the more massive proton.

5. An electron (mass = 0.511 MeV/c^2) and a proton (mass = 938 MeV/c^2) have the same DeBroglie wavelength. Which of the following statements is true?

   a) The electron's momentum is greater than that of the proton
   b) The proton's momentum is greater than that of the electron
   c) The electron's total energy equals that of the proton
   d) The electron's total energy is greater than that of the proton
   → e) None of the other statements are true

6. A particle in an infinite square well has an energy of 8 eV in the FIRST EXCITED state. What are its energies in the GROUND state and the SECOND EXCITED state respectively?

   a) 4 eV and 12 eV
   b) 4 eV and 36 eV
   → c) 2 eV and 18 eV
   d) $\sqrt{8}$ eV and 16 eV
   e) $\sqrt{8}$ eV and 64 eV

$E_n = \frac{n^2\pi^2\hbar^2}{2mL^2}$ and $E_2 = 8\,eV$.

So $E_1 = \frac{(1)^2}{(2)^2} E_2 = \frac{1}{4} E_2 = 2\,eV$.

$E_3 = \frac{(3)^2}{(2)^2} E_2 = \frac{9}{4}(8) = 18\,eV$.

7. Which of the following are possible sets of quantum numbers for an electron in an atom?

   - I: $n = 4,\ l = 2,\ m_l = -2,\ m_s = -1/2$
   - II: $n = 3,\ l = 3,\ m_l = 0,\ m_s = +1/2$ — False because it has $l = n$.
   - III: $n = 3,\ l = 0,\ m_l = 0,\ m_s = -1/2$

   $l = 0, 1, \ldots, (n-1)$.

   a) I is possible; II and III are not
   b) None of the three are possible
   → c) I and III are possible; II is not
   d) All three are possible
   e) III is possible; I and II are not

8. An electron is in a state of zero orbital angular momentum. What are the possible angles that its spin angular momentum vector can make with the z-axis, which could be the direction of an external magnetic field?

   → a) 55° and 125°
   b) 45° and 135°
   c) 60° and 120°
   d) 0° and 180°
   e) 90° only

$S = \sqrt{s(s+1)}\,\hbar = \sqrt{\frac{1}{2}\cdot\frac{3}{2}}\,\hbar = \frac{\sqrt{3}}{2}\hbar$.

$S_z = m_s\hbar = \pm\frac{1}{2}\hbar$

$\cos\theta = \frac{S_z}{S} = \frac{\pm\frac{1}{2}\hbar}{\frac{\sqrt{3}}{2}\hbar} = \pm\frac{1}{\sqrt{3}}$,

$\theta = 55^\circ$ AND 135°.

9. A certain subshell can have a maximum of 10 electrons. Which of the following could that subshell be?

a) 3s
b) 5p
c) 5g
d) 4f
→ e) None of the other answers

MAXIMUM subshell population $= 2(2\ell+1) = 10$
$\Rightarrow \ell = 2$, i.e. a "d" subshell.

10. Consider a complex atom in a state in which the spin quantum number is 2, and the orbital angular momentum quantum number is 3. What are all the possible values of the total angular momentum quantum number?

→ a) 1, 2, 3, 4, 5
b) 1 only
c) 5 only
d) 1 and 5
e) 1, 3, 5

$J = |L-S|, \ldots, (L+S)$
$= |3-2|, \ldots, (3+2)$
$= 1, 2, 3, 4, 5$

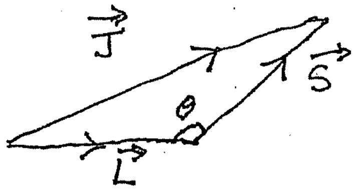
11. A neutral atom of silicon has 14 electrons. Which of the following is the electron configuration for the ground state of silicon?

a) $1s^2 2s^2 2p^6 2d^4$
→ b) $1s^2 2s^2 2p^6 3s^2 3p^2$
c) $1s^2 2s^2 3s^2 3p^6 3d^2$
d) $1s^2 1p^6 2s^2 2p^4$
e) None of the other answers

12. In the real universe, the first two inert gases occur at atomic numbers Z of 2 and 10. At what values of Z will the first two inert gases occur in a hypothetical universe in which the spin quantum number of the electron is 3/2 instead of 1/2?

a) 2 and 10
b) 4 and 16
→ c) 4 and 20
d) 6 and 30
e) None of the other answers

$s = \frac{3}{2} \Rightarrow m_s = -\frac{3}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{3}{2}$.
Four values of m_s.
So: first inert gas: $1s^4$
second inert gas: $1s^4 2s^4 2p^{12}$.



$n = 4,\ l = 3,\ s = 1,\ j = 4.$

$J^2 = L^2 + S^2 - 2LS\cos\theta$

13. An atom is in a 4^3F_4 state. For this state, what is the angle between the orbital and spin angular momentum vectors WITHIN the triangle for vector addition of these angular momenta?

    a) About 35°
    b) About 45°
    c) About 60°
    d) Exactly 90°
    → e) About 128°

$j(j+1) = l(l+1) + s(s+1) - 2\sqrt{l(l+1)}\sqrt{s(s+1)}\cos\theta$

$4(5) = 3(4) + 1(2) - 2\sqrt{12}\sqrt{2}\cos\theta$

$\Rightarrow \cos\theta = -.612 \Rightarrow \theta = 128^\circ.$

14. In the preceding problem, suppose the atom in the 4^3F_4 state is placed in a weak magnetic field B. What will be the spacing in energy between adjacent magnetic substates?

    a) $0.67\mu_B B$
    → b) $1.25\mu_B B$
    c) $1.50\mu_B B$
    d) $2.00\mu_B B$
    e) $1.33\mu_B B$

$\Delta E = \Delta m_j\, g\, \mu_B B$ where $\Delta m_j = 1.$

$g = 1 + \dfrac{j(j+1) + s(s+1) - l(l+1)}{2j(j+1)}$

$= 1 + \dfrac{4(5) + 1(2) - 3(4)}{2(4)(5)} = 1.25$

15. The outer electrons in Silicon have the configuration $3s^2 3p^2$. For Aluminum it's $3s^2 3p^1$, and for Indium it's $5s^2 5p^1$. If Silicon is doped with Aluminum or Indium, what kinds of semiconductors (if any) are formed?

    a) n-type for Aluminum, p-type for Indium
    b) p-type for Aluminum, n-type for Indium
    → c) p-type for both
    d) p-type for Aluminum, neither for Indium because the outer electrons are not in the $n = 3$ shell
    e) n-type for both

Dopant is short an electron in both cases, so there is a positive hole in acceptor levels.

16. A hydrogen atom undergoes a transition from a state of quantum number n to a state of quantum number $(n-4)$. The Bohr radius of the atom drops by a factor of nine as a result of the transition. What is the value of n?

    a) 13
    b) 9
    c) 5
    → d) 6
    e) None of the other answers

$r = n^2 a_0 \qquad r' = (n-4)^2 a_0$

$\dfrac{r}{r'} = 9 = \dfrac{n^2}{(n-4)^2} \Rightarrow 9n^2 - 72n + 144 = n^2$

$\Rightarrow 8n^2 - 72n + 144 = 0$

$\Rightarrow n^2 - 9n + 18 = 0$

Solutions: $n = 3$ or 6.

$n = 3$ is extraneous because it can't go from there to $n-4$.