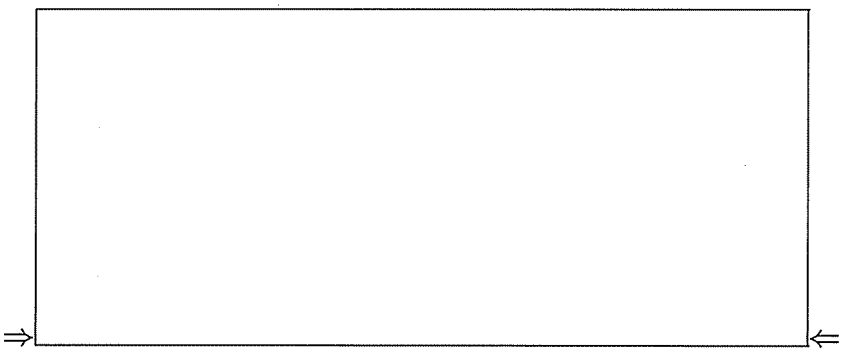
SOLUTIONS

Physics 273– Exam I
Wednesday, October 5, 2011
Prof. Mohan Kalelkar



Your name sticker with **exam code**

1. The exam will last from 1:45pm to 2:50pm. Use a # 2 pencil to make entries on the answer sheet. Enter the following id information now, before the exam starts.

2. In the section labelled NAME, enter your last name, then fill in the empty circle for a blank, then enter your first name, another blank, and finally your middle initial.

3. Under STUDENT # enter your 9-digit Student ID Number. Under COURSE enter 273. Under CODE enter the exam code given above.

4. During the exam, you may use pencils, a calculator, and **ONE** $8\frac{1}{2}'' \times 11''$ sheet of paper with formulas and notes.

5. There are 16 multiple-choice questions on the exam. For each question, mark only one answer on the answer sheet. There is no subtraction of points for an incorrect answer, so even if you cannot work out the answer to a question, you should make an educated guess. At the end of the exam, hand in only the answer sheet. Retain this question paper for future reference and study.

6. Useful numerical constants are given on the next page. Before starting the exam, make sure that your copy contains the page of constants and all 16 questions. Bring your exam to the proctor if this is not the case.

Elementary charge $e = 1.6 \times 10^{-19}\ C$
1 electron volt $(eV) = 1.6 \times 10^{-19}\ J$
Speed of light $c = 3 \times 10^{8}\ m/s$
Planck's constant $h = 6.63 \times 10^{-34}\ J \cdot s = 1240\ nm \cdot eV/c$
$\hbar = h/2\pi$
Compton wavelength of electron $h/mc = 0.0024\ nm$
Ground-state energy of hydrogen $= -13.6\ eV$
Rydberg constant $R = 0.0109678\ nm^{-1}$
Avogadro's number $= 6.02 \times 10^{23}$ molecules/mole
Electron mass $= 9.11 \times 10^{-31}\ kg = 0.511\ MeV/c^2$
Proton mass $= 1.673 \times 10^{-27}\ kg = 938.3\ MeV/c^2$
Neutron mass $= 1.675 \times 10^{-27}\ kg = 939.6\ MeV/c^2$
Atomic mass unit $1\ u = 931.5\ MeV/c^2$
Powers of ten:

femto(f) 10^{-15}	pico(p) 10^{-12}	nano(n) 10^{-9}	micro(μ) 10^{-6}	milli(m) 10^{-3}
centi(c) 10^{-2}	kilo(k) 10^{+3}	Mega(M) 10^{+6}	Giga(G) 10^{+9}	Tera(T) 10^{+12}

1. A spaceship of proper length 50 m is moving away from the earth at a speed of 0.8c. According to observers in the ship, their journey takes 6.0 hours. According to observers on earth, what is the length of the ship, and how long does the journey take?

a) 83 m; 10 *hours*
b) 30 m; 3.6 *hours*
c) 83 m; 3.6 *hours*
d) 50 m; 3.6 *hours*
→ e) 30 m; 10 *hours*

$\gamma = \frac{1}{\sqrt{1-v^2/c^2}} = \frac{1}{\sqrt{1-(.8)^2}} = 1.67.$

Ship measures proper length and time.

$L_E = \frac{50}{1.67} = 30\,m; \quad t_E = (1.67)(6.0) = 10\ hr.$

2. A truck of proper length 20 m approaches a bridge of proper length 18 m, which is at rest on the earth. According to observers on earth, the truck and the bridge are equal in length. What is the length of the bridge as measured by the truck driver?

→ a) About 16.2 m
b) About 18.0 m
c) About 19.8 m
d) About 7.9 m
e) About 10.0 m

$18 = \frac{20}{\gamma} \Rightarrow \gamma = \frac{20}{18}.$

Also, $L_T = \frac{18}{\gamma} = \frac{(18)(18)}{20} = 16.2\,m.$

3. A spaceship is moving *away* from earth at 0.3c. Its rebellious commander fires a torpedo *back towards* the earth at a speed of 0.5c relative to the ship. What is the speed of the torpedo relative to earth?

a) About 0.94c
b) About 0.80c
c) About 0.70c
→ d) About 0.24c
e) About 0.17c

K = earth; K' = ship. Then $v = .3c$, $u_x' = -.5c$.

$u_x = \frac{u_x' + v}{1 + \frac{v u_x'}{c^2}} = \frac{-.5c + .3c}{1 + \frac{(.3c)(-.5c)}{c^2}} = -.24c.$

4. A student steps into a library at Rutgers. 150 μs later (as measured on earth), another student steps into a library at Columbia University, located 60 km from Rutgers. Find the speed of a reference frame, moving along the line from Rutgers to Columbia, in which these two events are simultaneous.

→ a) About 0.75c
b) About 0.80c
c) About 0.85c
d) About 0.90c
e) About 0.95c

K = earth. $\Delta x = 60\,km$, $\Delta t = 150\,\mu s$.

$\Delta t' = \gamma\left(\Delta t - \frac{v\Delta x}{c^2}\right)$

$0 = \gamma\left[150\times10^{-6} - \frac{v(60\times10^3)}{(3\times10^8)^2}\right]$

$\Rightarrow v = 2.25\times10^8\ m/s$

$= .75c.$

$$\Delta t' = \gamma\left(\Delta t - \frac{v\Delta x}{c^2}\right)$$

5. In an inertial reference frame K, two events occur on the x-axis separated in time by Δt and in space by Δx. In another inertial reference frame K', moving in the x-direction relative to K, the two events could occur at the same time under which, if any, of the following conditions?

   a) For any values of Δx and Δt
   → b) Only if $|\Delta x/\Delta t| > c$
   c) Only if $|\Delta x/\Delta t| < c$
   d) Only if $|\Delta x/\Delta t| = c$
   e) Under no conditions

"Same time in K'" means $\Delta t' = 0$.

So $\Delta t = \frac{v\Delta x}{c^2} \Rightarrow \frac{\Delta x}{\Delta t} = \frac{c^2}{v} > c.$

6. A particle of mass M has a momentum of $2Mc$. If the momentum is then doubled, by what factor does its total energy increase?

   a) About 1.41
   → b) About 1.84
   c) About 2.00
   d) About 4.00
   e) None of the other answers

$E = \sqrt{(2Mc)^2c^2 + M^2c^4} = \sqrt{5}\,Mc^2.$

$E' = \sqrt{(4Mc)^2c^2 + M^2c^4} = \sqrt{17}\,Mc^2$

$E'/E = \sqrt{\frac{17}{5}} = 1.84$

7. A particle of mass M has a kinetic energy of $3Mc^2$. In which of the following ranges does its speed v lie?

   a) $v < 0.80c$
   b) $0.80c \le v < 0.85c$
   c) $0.85c \le v < 0.90c$
   d) $0.90c \le v < 0.95c$
   → e) $v \ge 0.95c$

$E = 3Mc^2 + Mc^2 = 4Mc^2$

$p = \sqrt{(4Mc^2)^2 - M^2c^4} = \sqrt{15}\,Mc$

$v = \frac{pc^2}{E} = \frac{\sqrt{15}}{4}c = 0.97c$

8. A particle of mass m has a momentum of $2mc$. What is its kinetic energy?

   a) About $2.24mc^2$
   b) About $2.00mc^2$
   → c) About $1.24mc^2$
   d) About mc^2
   e) About $3.00mc^2$

$E = \sqrt{(2mc)^2c^2 + m^2c^4} = \sqrt{5}\,mc^2$

$K = \sqrt{5}\,mc^2 - mc^2 = 1.24mc^2$

9. A particle of mass 498 MeV/c^2 is at rest. It spontaneously breaks into two fragments of equal mass, moving in opposite directions at equal speeds of 0.83c. What is the mass of either fragment? (Note that "mass" means "rest mass").

→ a) 139 MeV/c^2
b) 167 MeV/c^2
c) 249 MeV/c^2
d) 278 MeV/c^2
e) 334 MeV/c^2

Energy conservation: $Mc^2 = \gamma mc^2 + \gamma mc^2$

$\Rightarrow m = \frac{M}{2\gamma}$. $\gamma = \frac{1}{\sqrt{1-(.83)^2}} = 1.79.$

$m = \frac{498}{(2)(1.79)} = 139\ MeV/c^2.$

10. A photon strikes an electron of mass m that was initially at rest, creating an electron-positron pair. The photon is destroyed in the collision, and the final state consists of the positron and two electrons, all of which have the same mass m. They also have equal speeds and all move along the initial direction of the photon. What was the photon's energy?

$p_e = \frac{1}{3} p_\gamma = \frac{1}{3}\frac{E_\gamma}{c}.$

a) mc^2
b) $2mc^2$
c) $3mc^2$
→ d) $4mc^2$
e) $5mc^2$

Energy cons.: $E_\gamma + mc^2 = 3\sqrt{\left(\frac{E_\gamma}{3c}\right)^2 c^2 + m^2c^4}$

$E_\gamma^2 + 2mc^2E_\gamma + m^2c^4 = 9\left(\frac{E_\gamma^2}{9} + m^2c^4\right) = E_\gamma^2 + 9m^2c^4$

$\Rightarrow 2mc^2E_\gamma = 8m^2c^4 \Rightarrow E_\gamma = 4mc^2.$

11. An astronomical object emits light of wavelength 121.5 nm. An observer on Earth measures the wavelength of the light received from the object to be 607.5 nm. What is the velocity of the object?

a) About 3.0×10^8 m/s away from Earth
b) About 2.4×10^8 m/s away from Earth
→ c) About 2.8×10^8 m/s away from Earth
d) About 2.4×10^8 m/s toward Earth
e) About 2.8×10^8 m/s toward Earth

$\sqrt{\frac{1+\beta}{1-\beta}} = \frac{607.5}{121.5} = 5$

$1+\beta = 25(1-\beta) \Rightarrow \beta = .923$

$v = (.923)(3\times10^8) = 2.8\times10^8\ m/s.$

12. A laser is emitting 10^{17} photons every second, all at a wavelength of 633 nm. What is the laser's power output?

a) About 0.75 W
→ b) About 0.03 W
c) About 12 W
d) About 0.20 W
e) About 4.1 W

$P = N\frac{hc}{\lambda}$

$= (10^{17})\frac{(6.63\times10^{-34})(3\times10^8)}{633\times10^{-9}} = .03\ W.$

13. The threshold wavelength for photoemission in silver is 262 nm. For what wavelength will the photoelectric stopping potential be 1.36 V?

→ a) About 204 nm
b) About 175 nm
c) About 368 nm
d) About 912 nm
e) About 127 nm

$\phi = \frac{hc}{\lambda_0} = \frac{1240\ nm\cdot\frac{eV}{c}\cdot c}{262\ nm} = 4.73\ eV,$

$E_\gamma = \phi + eV_0 = 4.73 + 1.36 = 6.09\ eV.$

$\lambda = \frac{hc}{E_\gamma} = \frac{1240}{6.09} = 204\ nm.$

14. In a photoelectric effect experiment, which of the following changes would cause the stopping potential to INCREASE?

- I: Increasing the frequency of the light
- II: Increasing the intensity of the light
- III: Using a different metal with a smaller work function

a) I only
b) I and II, but not III
c) II and III, but not I
→ d) I and III, but not II
e) All three would cause the stopping potential to increase

$h\nu = \phi + eV_0$

15. A photon moving at speed c makes a Compton collision with a free electron at rest. After the collision, the scattered photon travels at an angle θ relative to the direction of the incident photon. What is the speed of the scattered photon?

a) $c\ \cos\theta$
b) $c\ (1-\cos\theta)$
c) $c\ \sin\theta$
d) $c\ (1-\sin\theta)$
→ e) None of the other answers

Speed of light is c !

16. A photon of wavelength 0.0016 nm undergoes Compton scattering off a free electron initially at rest. The scattered photon emerges at an angle of 60° with respect to the incident photon direction. What kinetic energy does the electron acquire?

a) About 1.22 MeV
→ b) About 0.33 MeV
c) About 0.77 MeV
d) About 0.44 MeV
e) About 1.03 MeV

$E = \frac{hc}{\lambda} = \frac{1240\times10^{-6}}{.0016} = .775\ MeV.$

$\lambda' = \lambda + \frac{h}{mc}(1-\cos\theta) = .0016 + .0024(1-\cos 60^\circ)$

$= .0028\ nm.$ So $E' = \frac{1240\times10^{-6}}{.0028} = .443\ MeV.$

Then $K = E - E' = .775 - .443 = .332\ MeV.$