General Theory of Relativity
Inertial Frames

How do you know if you are in an **inertial frame**?

Answer: to see if Newton’s Laws are still working.

For example: Is the surface of the Earth an inertial frame?

No, because the Earth is **rotating**. What kind of experiment would you need to conduct to test this?
Well, if you shot a cannon ball a great distance, you would notice that it would be deflected. This is the Coriolis effect, and it occurs because the Earth’s surface is not an inertial frame.
Suppose you were inside a rocket ship. What experiment could you perform to tell the difference between sitting at rest on the Earth and accelerating upward with an acceleration $g$?
**Principle of equivalence:** the effects of acceleration and gravitation cannot be distinguished in a closed lab.

*Newton’s 2nd Law:* \[ F = m_I a \]

Here \( m_I \) is the **inertial mass**.

*Law of Gravity:* \[ F = \frac{G m_G M}{R^2} \]

Here \( m_G \) is the **gravitational mass**.

That \( m_I = m_G \) is not a trivial statement!
Consider a rocket in motion **accelerating upwards**. Suppose a beam of light enters from the left.
In the frame of the beam of light, the light is traveling in a straight path.

In the frame of the rocket, the light appears to bend downward.
But gravitation and acceleration are equivalent concepts. Therefore, light should be bent by gravity. This was confirmed in 1919.

What is a straight line? It’s the shortest distance between two points, i.e. it’s the path that light takes. This picture necessitates non-Euclidean geometry.
**FIGURE 15.17** Path of signal between Earth and Venus in curved spacetime. The actual path (solid line) is slightly longer than the Euclidian direct path. (Diagrams like this, called embedding diagrams, represent one way of picturing a two-dimensional slice of four-dimensional spacetime.)
Mercury’s perihelion should advance 531 seconds of arc per century. It actually advances 574” per century. The extra 43” per century is what was predicted by general relativity.
A binary pulsar system was discovered by Hulse and Taylor in 1974. A pulsar and neutron star orbit each other with a period of ~8 hours at ~1/1000 the speed of light.

With each rotation, the orbit shrinks (~1 cm per day) because of gravitational radiation as predicted by GR.
Consider a photon in a gravitational well. As it falls into the well, it gains energy. As it climbs out of the well, it loses energy. The difference in energy should be given by:

\[ \Delta E = -GmM \left( \frac{1}{r_1} - \frac{1}{r_2} \right) \]

where \( m \) is the “mass” of the photon. For sake of argument, take it to be \( E/c^2 \).

\[ \frac{\Delta E}{E} = \frac{\Delta \nu}{\nu} = -\frac{\Delta \lambda}{\lambda} = -\frac{GM}{c^2} \left( \frac{1}{r_1} - \frac{1}{r_2} \right) \]

Therefore, there is a **frequency shift** between distances \( r_1 \) and \( r_2 \).
Example 1: Redshift of light from a neutron star.

\[ r_1 = \text{radius of the star} = R \]

\[ r_2 = \text{distance to earth} = \infty \]

\[
\frac{\Delta \lambda}{\lambda} = \frac{GM}{c^2} \left( \frac{1}{r_1} - \frac{1}{r_2} \right) = \frac{GM}{c^2 R}
\]

Example 2: Blueshift of light come to us from a satellite at a height \( h \).

\[ r_2 = \text{radius of the earth} = R_E \]

\[ r_1 = \text{distance to satellite from the Earth’s center} = R_E + h \]

\[
\frac{\Delta \lambda}{\lambda} = \frac{GM}{c^2} \left( \frac{1}{R_E + h} - \frac{1}{R_E} \right) = \frac{GMh}{c^2 R_E(R_E + h)} \approx \frac{GMh}{c^2 R_E^2}
\]
How massive would a body need to be in order to infinitely redshift a photon?

Infinite redshift means the change in the wavelength is equal to the original wavelength:

\[ \Delta \lambda = \lambda \]

So, assuming the observer is infinitely far away:

\[ \frac{\Delta \lambda}{\lambda} = 1 = \frac{GM}{c^2} \left( \frac{1}{R_S} - \frac{1}{\infty} \right) = \frac{GM}{c^2 R_S} \quad \rightarrow \quad R_S = \frac{GM}{c^2} \]

If we were to do this calculation correctly with general relativity, we recover an extra factor of 2:

\[ R_S = 2 \frac{GM}{c^2} \]

This is the Schwarzschild radius. If a massive object has radius smaller than this, it becomes a black hole.
Consequences of the Principle of Equivalence

- gravity deflects the path of light
- non-Euclidean geometry is needed to describe spacetime
- spacetime is curved. Matter (and energy) warp spacetime. The effects of “gravity” are due to motion in a curved spacetime.
- Gravity causes a redshift of light, analogous to the doppler shift.
- When a massive body approaches the Schwarzschild radius $R_s = 2GM/c^2$, the light gets redshifted infinitely.
https://www.youtube.com/watch?v=s06_jRK939I