Cosmology
Doppler shift of light from other galaxies tell us their speed relative to us:

$$\lambda = \lambda_0 \sqrt{\frac{1 + \beta}{1 - \beta}} \quad \text{where} \quad \beta \equiv \frac{v}{c}$$

Usually discuss in terms of redshift ($z$):

$$z \equiv \frac{\lambda - \lambda_0}{\lambda_0} = \frac{\lambda}{\lambda_0} - 1 = \sqrt{\frac{1 + \beta}{1 - \beta}} - 1$$

Most galaxies are red-shifted from us (particularly the distant ones)

This means that most galaxies are receding from us.
Hubble’s Law

The velocity that a galaxy is receding from us is proportional to its distance $R$

$$v = H_0 R$$

The constant of proportionality is Hubble’s constant $H_0$

$H_0$ is about 70 km/s/Mpc = 0.021 m/s/light-year

1 Mpc (mega-parsec) = $3.26 \times 10^6$ light years

= $3.1 \times 10^{22}$ m

(Beware: Hubble’s constant is not really constant. It’s been decreasing with time.)
$H_0 = 68 \text{ km/s Mpc}$

Virgo Cluster
Example: A galaxy is located 3000 Mpc from us.

How fast is it receding from us?

\[
\beta = \frac{v}{c} = \frac{H_0 R}{c} = \frac{(71 \text{ km/s/Mpc}) \cdot (3000 \text{ Mpc})}{3 \times 10^5 \text{ km/s}} = 0.71
\]

What is the redshift?

\[
z = \sqrt{\frac{1 + \beta}{1 - \beta}} - 1 = \sqrt{\frac{1.71}{0.29}} - 1 \approx 1.43
\]

What is the wavelength we would measure for the H\(_\alpha\) line (656 nm) emitted by the galaxy?

\[
\Delta \lambda = 1.43\lambda_0 = 938 \text{ nm}
\]

\[
\lambda = \lambda_0 + \Delta \lambda = 656 \text{ nm} + 938 \text{ nm} = 1594 \text{ nm}
\]
At what distance are galaxies receding from us at the velocity of light?

\[
\frac{v}{c} = 1 = \frac{H_0 R}{c} \quad \rightarrow \quad R = \frac{c}{H_0} = \frac{3 \times 10^5 \text{ km/s}}{71 \text{ km/s/Mpc}} \approx 4.2 \text{ Gpc}
\]

How much time did it take for the light to travel?

4.2 Gpc = 13.7 billion light years.

That’s roughly how old we think the Universe is.
Let’s suppose that the expansion started at $t=0$. What happened since then?

At $t \sim 10^{-43}$ s, our understanding of physics is very speculative (requires quantum gravity, and all that).

At $t \sim 10^{-34}$ s, the Universe has a temperature of $10^{26}$ K. Photons and matter are in equilibrium.

\[
\gamma + \gamma \iff \text{particle + it's antiparticle}
\]

(recall Boltzmann’s constant $k=8.6 \times 10^{-11}$ MeV/K)
At $t \sim 10^{-6}$ s, $T \sim 10^{13}$ K.

Matter exceeds antimatter by about one part in $10^9$. $kT \sim 1$ GeV, so no more nucleon pair production.

Quarks can now combine to form hadrons. Nucleons and antinucleons start to annihilate each other.

There is a slight asymmetry (part in $\sim 10^9$) between matter and anti-matter (because of CP violation) so more matter exists than anti-matter.

This process is called **baryogenesis**.
At $t \sim 1 \text{s}$, $T=10^{10} \text{ K}$

$kT < 1 \text{ MeV}$, so no more $e^+e^-$ pair production. Electrons and positrons start to annihilate each other without replenishment.

The universe consists of a plasma of nuclei, electrons and photons; temperatures remain too high for the binding of electrons to nuclei.

At $t \sim 10 \text{ s}$, $T\sim10^9 \text{ K}$

$kT\sim0.1 \text{ MeV}$, so the photons are no longer energetic enough to break up nuclei, but protons can combine with neutrons.

**Nucleosynthesis** begins.
At \( t \approx 300,000 \) years, \( T \approx 10^3 \) K

\[ kT \approx 0.1 \text{ eV}, \text{ so photons can no longer keep electrons from binding with nuclei.} \]

Photons stream freely. This is the source of the cosmic microwave background (CMB).

The CMB is *microwave* and not \( 10^3 \) K because the Universe has been expanding (cooling) since then, causing photons to redshift.
How much matter does there need to be in the Universe so that the expansion is balanced exactly by the gravitational attraction?

Consider a galaxy with mass $m$ moving at a velocity $v$. It is being attracted by the rest of the Universe with mass $M$.

\[
\frac{1}{2}mv^2 - G\frac{mM}{R} = 0
\]

Plugging in Hubble’s Law ($v=H_0R$), we get

\[
\frac{1}{2}mH_0^2R^2 - G\frac{mM}{R} = 0
\]

Solving for $M$ we get

\[
M = \frac{H_0^2R^3}{2G} \quad \rightarrow \quad \rho_{cr} = \frac{M}{\frac{4}{3}\pi R^3} = \frac{3H_0^2}{8\pi G} \approx 9 \times 10^{-27} \frac{\text{kg}}{m^3}
\]
It turns out that:
\[ \Omega(\text{luminous matter}) \approx 0.04 \]
\[ \Omega(\text{dark matter}) \approx 0.23 \]
\[ \Omega(\text{dark energy}) \approx 0.73 \]
\[ \Omega(\text{total}) \approx 1.0 \]

\( \Omega_M = \frac{\rho_M}{\rho_{cr}} \) matter fraction of the Universe
\( \Omega_\Lambda = \frac{\rho_\Lambda}{\rho_{cr}} \) dark energy fraction of the Universe