Fission and Fusion
As you may know by now, natural Uranium is about 99.3% U-238 and 0.7% U-235.

\[ ^1_0n + ^{235}_{92}U \rightarrow ^{144}_{56}\text{Ba} + ^{89}_{36}\text{Kr} + ^1_0n + ^1_0n + ^1_0n \]

The energy release in the above process is **about 200 MeV**. (compares to about a **few eV per atom** with combustion).

Remember where U is on the binding energy curve?
Now since there are three neutrons produced per reaction, each neutron might hit another Uranium nucleus, producing yet more neutrons...

\[ \frac{1}{0}n + \frac{235}{92}U \rightarrow \frac{144}{56}Ba + \frac{89}{36}Kr + \frac{1}{0}n + \frac{1}{0}n + \frac{1}{0}n \]

This is the basis of a chain reaction.
1 neutron in the first stage
3 neutrons in the second stage
9 neutrons in the third stage
27 neutrons in the fourth stage
81 neutrons in the fifth stage
...

Of course, this is idealized. Fission isn’t nearly so efficient...
What could make fission inefficient?

1. Competing reactions, eg:
\[ _1^0 n + _{92}^{235}U \rightarrow _{54}^{140}Xe + _{38}^{94}Sr + _0^1 n + _0^1 n \]

2. Capture of neutrons by U-238, instead of U-235, followed by non-fission decay

3. Leakage of neutrons that don’t interact

Each of these can prevent a **self-sustaining** chain reaction.
While nothing can be done about competing reactions (that’s determined by quantum mechanics), one can deal with the other issues.

Can **slow down** neutrons to avoid U-238 capture. Often use D$_2$O or graphite as a “moderator” to slow them down.
Of course, a “critical mass” of fissile material is needed for a chain reaction.

If each neutron leads to $N$ more neutrons on average then the following holds

$N<1$: subcritical (no chain reaction)
$N=1$: critical (nuclear reactor goal)
$N>1$: supercritical (this is what’s needed for a weapon)

Reactors use control rods made of Cd-113 to keep $N$ close to 1.

The problems for reactors are not supercritical reactions, but core meltdown (which can lead to a loss of containment) and waste disposal (which needs to be kept securely confined for 1000’s of years.)
Fusion of hydrogen into helium releases energy: mass converted to kinetic energy:

\[
\begin{align*}
\frac{2}{1}H + \frac{2}{1}H &\rightarrow \frac{3}{2}He + \frac{1}{0}n + 3.3 \text{ MeV} \\
\frac{2}{1}H + \frac{3}{1}H &\rightarrow \frac{4}{2}He + \frac{1}{0}n + 17.6 \text{ MeV}
\end{align*}
\]

Advantages over fission as a fuel:

1. absence of radioactive waste
2. plentiful supply of deuterons compared to U-235

Problems:

Fusion is very difficult to induce: must overcome the Coulomb repulsion. Practically, not yet efficient (more energy required to induce than is produced).
Proton-Proton Cycle in the Sun
Net result of pp-chain

Convert four protons into: one He-4, 2e\(^+\), 2\(\gamma\), 2\(\nu\) and 26.7 MeV of kinetic energy!

Fission gives \(~1\) MeV/nucleon

\((0.09\%\ of\ mass\ converted\ to\ kinetic\ energy)\)

Fusion gives \(~7\) MeV/nucleon

\((0.66\%\ of\ mass\ converted\ to\ kinetic\ energy)\)
Example

Sun's power output = 4\times10^{26} \text{ W} = 2\times10^{39} \text{ MeV/s}

Question: if 90\% of this energy is coming from the pp cycle, how many protons are fused every second?

Q=26.7 \text{ MeV}, so:

\[
\frac{\text{reactions}}{\text{second}} = \frac{0.90 \cdot 2 \times 10^{39}}{26.7} = 6.7 \times 10^{27}
\]

So the sun is burning 4\times6.7\times10^{27} = 3\times10^{28} protons every second!
Example

Sun releases $1.3 \times 10^{38}$ neutrinos/second

Earth-Sun distance is $1.5 \times 10^{11}$ m

What is the neutrino flux at the Earth’s surface?

\[
\frac{1.3 \times 10^{38}}{4\pi \cdot (1.5 \times 10^{11} \text{ m})^2} = 5 \times 10^{14} \text{ neutrinos/m}^2 \cdot \text{s}
\]
Recall way back when...

The temperature of the sun ~5500 K, that is photons have energy of a few eV.

But in the solar fusion process, the kinetic energy of the photons is ~few MeV.

The sun is very dense, so photons scatter many times before they reach the surface of the sun. It takes ~a million years for a photon produced at the core to reach the surface.

On the other hand, neutrinos don’t interact at all. They only take 8 minutes to reach the surface of the earth!
\[
\begin{align*}
\frac{1}{2}H + \frac{1}{2}H & \rightarrow \frac{3}{2}H + e^+ + \nu_e \\
\frac{3}{2}H + \frac{1}{2}H & \rightarrow \frac{3}{2}He + \gamma \\
69\% & \quad 31\% \\
\frac{3}{2}He + \frac{3}{2}He & \rightarrow \frac{3}{2}He + 2\frac{1}{2}H \\
\frac{3}{2}He + \frac{3}{2}He & \rightarrow \frac{7}{4}Be + \gamma \\
(PP\ I) & \\
\frac{7}{4}Be + e^- & \rightarrow \frac{7}{3}Li + \nu_e \\
99.7\% & \quad 0.3\% \\
\frac{7}{3}Li + \frac{1}{2}H & \rightarrow 2\frac{1}{2}He \\
\frac{7}{4}Be + \frac{1}{2}H & \rightarrow \frac{8}{5}B + \gamma \\
(PP\ II) & \\
\frac{8}{5}B & \rightarrow \frac{8}{4}Be + e^+ + \nu_e \\
\frac{8}{4}Be & \rightarrow 2\frac{1}{2}He \\
(PP\ III)
\end{align*}
\]
“CNO” cycle dominates in stars with mass 1.3 times the sun

\[ 4\text{H} + 2\text{e}^- \rightarrow \text{He} + 2\nu_e + 6\gamma + 26.7 \text{ MeV} \]

\[ ^{12}\text{C} + ^1\text{H} \rightarrow ^{13}\text{N} + \gamma + 1.95 \text{ MeV} \]
\[ ^{13}\text{N} \rightarrow ^{13}\text{C} + e^+ + \nu_e + 1.20 \text{ MeV (half-life of 9.965 minutes)} \]
\[ ^{13}\text{C} + ^1\text{H} \rightarrow ^{14}\text{N} + \gamma + 7.54 \text{ MeV} \]
\[ ^{14}\text{N} + ^1\text{H} \rightarrow ^{15}\text{O} + \gamma + 7.35 \text{ MeV} \]
\[ ^{15}\text{O} \rightarrow ^{15}\text{N} + e^+ + \nu_e + 1.73 \text{ MeV (half-life of 122.24 seconds)} \]
\[ ^{15}\text{N} + ^1\text{H} \rightarrow ^{12}\text{C} + ^4\text{He} + 4.96 \text{ MeV} \]

- **Proton**
- **Gamma Ray**
- **Neutron**
- **Neutrino**
- **Positron**
All known matter particles are made up of some combination of quarks and leptons interacting via force carrying bosons.
Electromagnetic Force

All “charged” particles interact under this force

Field description discovered by Maxwell

Force is mediated by the photon

\[ e^- \rightarrow \gamma \]
Strong Force (~10)

- Keeps the nucleons in the atom together
- Force is mediated by the gluon
- All “colored” particles (e.g. quarks/gluons) interact via the strong force
Weak Force

Governs beta decays of the nucleus
Both leptons and quarks interact under this force
Force is mediated by “heavy photons” ($W^+/W^-/Z^0$)