Particles and Waves
When an electromagnetic wave (with wavelength $\lambda$) approaches crystal lattice with spacing (of size $d$), a diffraction pattern emerges:

Maxima in Bragg Diffraction occur at angles that satisfy:

$$2d \sin \theta = n \lambda$$

where $n = 1, 2, \ldots$
Example: if the \( n=1 \) peak occurs at \( 9^\circ \), how many peaks will there be?

\[
2d \sin \theta = n \lambda \quad \rightarrow \quad \frac{n \lambda}{2d} = \sin \theta
\]

Since \( \sin \theta \) is bounded by 1, we have

\[
\left| \frac{n \lambda}{2d} \right| \leq 1 \quad \rightarrow \quad n \leq \frac{2d}{\lambda}
\]

Plugging the given values into the Bragg equation, we have

\[
2d \sin 9^\circ = 1 \cdot \lambda \quad \rightarrow \quad \frac{2d}{\lambda} \approx 6.4
\]

Therefore there are at most 6 peaks.
Fundamental properties of waves: they can interfere with each other.
How a bullet behaves in a double-slit experiment:

The bullet’s behavior is particle-like.
How a wave behaves in a double-slit experiment:

A wave will pass through both slits simultaneously, resulting in an interference pattern. Light is wavelike.
What happens if you “shine” \textbf{electrons}?

But this seems \textbf{wavelike}, too...
But wait! The interference pattern emerges because a wave passes through both slits at the same time. But an electron (and a photon) is a particle, too.

So the interference pattern must result because multiple electrons/photons are passing through the slits and interfere with each other, right?
PRS Question: Suppose we limit the number of electrons so that only one electron passes the slits at a time. Which pattern do you get?

A) Pattern A (particle-like)
B) Pattern B (wavelike)
C) Neither
This was experimentally tested in 1961. It turns out that all particles are fundamentally wavelike. Even if you limit the electrons so that only one passes at a time, an interference pattern emerges. In 1999, buckyballs (C\textsubscript{60}) were shown to exhibit interference!

This means that a single electron must be simultaneously pass through both slits.

PRS Question: Suppose we limit the number of electrons so that only one electron passes the slits at a time. Which pattern do you get?

- A) Pattern A (particle-like)
- B) Pattern B (wavelike)
- C) Neither
De Broglie Waves (1924)

Fundamental relationship: all particles have wavelike properties with a wavelength inversely proportional to its momentum.

\[ \lambda = \frac{h}{p} \]

Experimental key: wavelength has to be larger than the slit size to see a diffraction pattern. If the wavelength is much smaller than the slit size, the pattern will look particle-like.
Why don’t bullets show diffraction?

Suppose bullet a weight 1 gram and travels at 500 m/s. What is the wavelength?

\[ p = mv = (10^{-3} \text{ kg}) \cdot (500 \text{ m/s}) = 0.5 \frac{\text{J} \cdot \text{s}}{\text{m}} \]

\[ \lambda = \frac{h}{p} = \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{0.5 \frac{\text{J} \cdot \text{s}}{\text{m}}} \approx 1.3 \times 10^{-33} \text{ m} \]

So the slit size should be \(~10^{18} \text{ times smaller than the size of the nucleus.}\)

Actually, it’s a lot more complicated than that... even if you make the slit sufficiently small, the bullet will never exhibit wavelike behavior. We’ll come back to that later.
What happens when you set up a detector to determine which slit the electron/photon passed through?

You destroy the interference pattern!
De Broglie equation quiz

\[ \lambda = \frac{h}{p} \]

PRS Question: A photon and an electron have the same total energy. Which has the greater wavelength?
A) Photon  
B) Electron  
C) Both the same
Recall Einstein’s relation:

$$E^2 = p^2 c^2 + m^2 c^4$$

$$\lambda = \frac{h}{p} = \frac{hc}{pc} = \frac{hc}{\sqrt{E^2 - m^2 c^4}}$$

The electron has **more mass** so its momentum $p$ is **lower** given the same energy $E$. Which makes its wavelength greater.

**PRS Question:** A photon and an electron have **the same total energy**. Which has the **greater** wavelength?

- A) Photon
- B) Electron
- C) Both the same