Exam Sunday April 9, 6:10-7:10+ on Busch

COME DOWN AND PLAY WITH THE ROLLING DISKS AND THE MAGNET ON THE RAMP

Electric fields and fixed charges

Coulomb’s law

$\mathbf{dE} = k \frac{dq}{r^2} \hat{r}$

Gauss’ law

closed surface

$\int_S \mathbf{E} \cdot d\mathbf{A} = \frac{q_{enc}}{\varepsilon_0}$

Magnetic fields and steady currents

Biot-Savart law

$\mathbf{dB} = \frac{\mu_0}{4\pi} \frac{i ds \times \hat{r}}{r^2}$

Ampere’s law – closed loop

$\int_C \mathbf{B} \cdot d\mathbf{s} = \mu_0 i_{enc}$

What happens when charges and currents change with time?
The DEMO

How can I get a current to flow in this coil?
Insert a battery into the loop
Potential difference across the ends –
Electric field along the wire
The DEMO

How can I get a current to flow in this coil?

Current induced in a loop by a moving bar magnet

Rate of motion
Relative motion
N/S towards/away
The DEMO

How can I get a current to flow in this coil?

Current induced in a loop by a moving bar magnet

Rate of motion
Relative motion
N/S towards/away

Changing magnetic field
Verify: current induced in a loop by changing current in nearby loop
Electric field lines form a closed loop

THIS NEVER HAPPENS IN ELECTROSTATICs!

Changing magnetic field creates an electric field in addition to and DIFFERENT IN CHARACTER from that created by electric charges
New law that relates CHANGES in magnetic field to created electric field

**FARADAY’S LAW**

\[ \int_C \vec{E} \cdot d\vec{s} = - \frac{d}{dt} \int_S \vec{B} \cdot d\vec{A} \]

line integral  
surface integral (magnetic flux)

Loop C is the boundary of the surface S
Magnetic flux through a surface

Uniform magnetic field $\vec{B}$

Planar loop – flat surface $A, \hat{n}$

Flux = $BA \cos \theta$

Units $1 \text{T m}^2 = 1 \text{ Wb (weber)}$
\( B(t) \) out of page increasing with time

\[ \Phi = B(t)A \]

\( \frac{d\Phi}{dt} \) is not zero

Area 0.2 m\(^2\)

\( B(t) = (0.1 \text{T/s}) t \)

\( \frac{d\Phi}{dt} = 0.02 \text{ T m}^2/\text{s} \)

Remember 1 T = 1 Ns/(Cm)

\( \frac{d\Phi}{dt} = 0.02 \text{ Nm/C} = 0.02 \text{ V} \)
The boundary of the surface $S$ is the closed loop $C$

$$\int_C \vec{E} \cdot d\vec{s}$$

Direction for line integral – if you walk along $C$ with normal vector to surface as up, you should go the direction that keeps the surface on your left

counterclockwise
B(t) out of page increasing with time

\[ \Phi = B(t)A \]

\( \frac{d\Phi}{dt} \) is not zero

\[ \int_C \vec{E} \cdot d\vec{s} \] is not zero

The current in the loop is not zero

Bulb demo
\( B(t) \) out of page increasing with time

\[ \Phi = B(t)A \]

\( \frac{d\Phi}{dt} \) is not zero

Area 0.2 m\(^2\)

\( B(t) = (0.1\, \text{T/s}) \, t \)

\( \frac{d\Phi}{dt} = 0.02 \, \text{V} \)

\[ \int_C \vec{E} \cdot d\vec{s} = -0.02 \, \text{V} \]

E field loops clockwise

\( \hat{n} \) points out of page
$B(t)$ out of page increasing with time

$\Phi = B(t)A$

d$\Phi$/dt is not zero

Area 0.2 m$^2$

$B(t) = (0.1 \text{T/s}) t$

$\frac{d\Phi}{dt} = 0.02 \text{ N m/C} = 0.02 \text{V}$

$\int_C \vec{E} \cdot d\vec{s} = -0.02 \text{V}$

$\hat{n}$ points out of page

wire of resistance $R$ made into a loop

E field loops clockwise

Current loops clockwise
Magnetic field at the center due to clockwise $i$ is into the page.

Magnetic flux due to current in loop is negative.
Magnetic field at the center due to counterclockwise $i$ is out of the page

Magnetic flux due to current in loop is **positive**
B(t) out of page increasing with time

\[ \Phi = B(t)A \]

Current flows in loop as if a battery were included

\[ \mathcal{E} = -\frac{d\Phi}{dt} \]

Why the minus sign?

direction of induced current: current flowing in loop creates an additional magnetic flux that OPPOSES \( \frac{d\Phi}{dt} \)

LENZ’S LAW
B(t) out of page and increasing with time. Which direction does the induced current flow? 
(A) Clockwise
(B) Counterclockwise
(C) The current is zero

The induced current produces a magnetic flux that OPPOSES THE CHANGE.
B(t) out of page and increasing with time

Clockwise
Force on the ring is inwards
Counterclockwise
Force on the ring is outwards

CAN DEMO
Simplest way to change the flux
Change the magnetic field

example 1: loop inside a solenoid with changing I

Uniform field inside solenoid
B(t) out of page and increasing with time. Which direction does the induced current flow?

(A) Clockwise
(B) Counterclockwise
(C) The current is zero

The induced current produces a magnetic flux that OPPOSES THE CHANGE.
OTHER WAYS TO CHANGE THE MAGNETIC FLUX THROUGH A LOOP

Move the loop into or out of a region with B field

Rotate the loop in a uniform field:
angle between B and n changes, so $\Phi$ changes

Change the size of the loop in a uniform field
Examples:
pull the loop closed
a moving metal rod on U-shaped wire
Move the loop into or out of a region with B field
B(r), surface S
Magnetic flux through a surface

\[ \Phi_B = \int \overrightarrow{B} \cdot d\overrightarrow{A} \]

Flat surface, B uniform on each part:

\[ \Phi = (\overrightarrow{B}_1 \cdot \hat{n})A_1 + (\overrightarrow{B}_2 \cdot \hat{n})A_2 \]
Move the loop into or out of a region with B field

\[ \Phi \text{ Negative increasing (less negative)} \]

Induced current is clockwise

Magnetic force on induced current in loop opposite to direction of motion
EDDY CURRENTS AND FORCES

Piece of metal moves in B field
Currents induced are called “eddy currents”
Force on induced currents opposite to the direction of motion

Lower resistivity means larger currents, larger forces

DEMOS

• ROLLING DISKS
• Magnet rolling down the ramp
• Magnet falling down the copper tube