Homework due Wednesday night 11:59 PM
Exam 2: Sunday April 9, 6:10-7:10 PM, on Busch
practice problems posted on web site
**start making your formula sheet now!**

**BEFORE CLASS:**
Play with magnets and moving charged particles
Lecture slides posted on main class web site

**CONGRATS ON 3/3 to** Alnadi, Balance, Alison Bansil, Buznitsky, Dias, Dormer, Z. Feng, Huang, Johnson, Kanagala, Kershaw, McKeown, Motyala, Ning, Parikh, Ravichandran, Rivera, Rondel, Scot, Smith, Suresh, Thuel, Tournoux, Yao
Chapter 28

Magnetic Field and Forces, part 2
Lodestone and bar magnets exert force on moving charged particles by creating a magnetic field $\vec{B}(r)$.

Bar magnet field: (near ends)

Force depends on $q$ and $\vec{v}$ and on the magnetic field $\vec{B}(r)$.
The magnetic force on a charged particle at $\vec{r}$

Cross product of two vectors $\vec{F} = q\vec{v} \times \vec{B}$

**Direction:**
- Perpendicular to direction of $\vec{v}$
- Perpendicular to direction of $\vec{B}$
- Normal to the plane defined by $\vec{v}$ and $\vec{B}$

Two choices – use right-hand rule to choose
- Thumb along $\vec{v}$; fingers along $\vec{B}$: out of palm
The magnetic force on a charged particle at $\vec{r}$

Cross product of two vectors $\vec{F} = q\vec{v} \times \vec{B}$

If the velocity and magnetic field vectors are given in component form, compute by expanding the terms

Example:

$\vec{v} = v_x \hat{i}$

$\vec{B} = B_x \hat{i} + B_y \hat{j}$

$\vec{v} \times \vec{B} = v_x \hat{i} \times (B_x \hat{i} + B_y \hat{j}) = v_x B_x (\hat{i} \times \hat{i}) + v_x B_y (\hat{i} \times \hat{j})$

$= v_x B_y \hat{k}$
i-clicker:
A particle of charge +1.0 C with velocity (in m/s)
\[ \vec{v} = 2\hat{i} + 3\hat{j} \]
moves through the uniform magnetic field (in T)
\[ \vec{B} = 3\hat{i} - 1.5\hat{j} \]
What is the magnitude of the magnetic force on the particle?

a) 12 N
b) 6.0 N
c) 4.5 N
d) 1.5 N
e) None of the above
i-clicker:
A particle of charge +1.0C with velocity (in m/s)
\( \vec{v} = 2\hat{i} + 3\hat{j} \)
moves through the uniform magnetic field (in T)
\( \vec{B} = 3\hat{i} - 1.5\hat{j} \)
What is the magnitude of the magnetic force on the particle?

a) 12 N  
b) 6.0 N  
c) 4.5 N  
d) 1.5 N  
e) None of the above
Motion of a charged particle $q$ in a uniform field $B$ $v_0$ perpendicular to the field

B out of the screen

Initially $F = q v_0 B$ downward
Motion of a charged particle $q$ in a uniform field $B$ $v_0$ perpendicular to the field

Initially $F = q v_0 B$ downward
Velocity stays constant: magnetic force does no work
$F$ stays $q v_0 B$ perpendicular to velocity
Motion of a charged particle $q$ in a uniform field $B$ $v_0$ perpendicular to the field

Initially $F = q v_0 B$ downward
Velocity stays constant: magnetic force does no work
$F$ stays $q v_0 B$ perpendicular to velocity
Motion of a charged particle $q$ in a uniform field $B$ perpendicular to the field $v_0$

Uniform circular motion
To get the radius of the circle, set the magnetic force equal to the centripetal force

$qv_0B = mv_0^2/r$ so $r = mv_0/qB$
Typical problem: $x = \frac{2mv_0}{qB}$

B into the screen: pos bends left, neg bends right

Table 28-4

| Question 10 |
|------------------|------------------|
| **Particle** | **Mass** | **Charge** | **Speed** |
| 1 | $2m$ | $q$ | $v$ |
| 2 | $m$ | $2q$ | $v$ |
| 3 | $m/2$ | $q$ | $2v$ |
| 4 | $3m$ | $3q$ | $3v$ |
| 5 | $2m$ | $q$ | $2v$ |
| 6 | $m$ | $-q$ | $2v$ |
| 7 | $m$ | $-4q$ | $v$ |
| 8 | $m$ | $-q$ | $v$ |
| 9 | $2m$ | $-2q$ | $3v$ |
| 10 | $m$ | $-2q$ | $8v$ |
| 11 | $3m$ | 0 | $3v$ |
Motion of a charged particle $q$ in a uniform field $B$
$v_{0x}$ perpendicular to the field
$v_{0z}$ parallel to the field

$B$ out of the screen

helix

Electron beam demo
Force on a current carrying wire in a uniform magnetic field

B points out of the screen
Magnetic force is to the right
Force on the charges is transmitted to the wire

\[ \vec{F}_B = i\vec{L} \times \vec{B} \]

\( \vec{L} \) is in the direction of the current

Electrons are the free charges here
The torque due to the tangential component of the force causes an angular acceleration around the rotation axis.

Torque along the rotation axis = $F_t r$ (positive)
i-clicker:
What is the sign of the torque on the rectangular loop rotating on the fixed axis with a uniform B field pointing to the right?

a) Positive (up along axis)
b) Negative (down along axis)
c) Zero
d) Just guessing
i-clicker:
What is the sign of the torque on the rectangular loop rotating on the fixed axis with a uniform B field pointing to the right?

a) Positive (up along axis)
b) Negative (down along axis)
c) Zero
d) Just guessing

Torque = 2 \((iB\alpha)(b/2) = iB(\alpha b) = iB \text{ area}\)
Fig. 28-18  The elements of an electric motor. A rectangular loop of wire, carrying a current and free to rotate about a fixed axis, is placed in a magnetic field. Magnetic forces on the wire produce a torque that rotates it. A commutator (not shown) reverses the direction of the current every half-revolution so that the torque always acts in the same direction.
Chapter 29

Currents Produce Magnetic Fields
Calculating the Magnetic Field due to a Current: Biot-Savart Law

This element of current creates a magnetic field at $P$, into the page.

\[ d\vec{B} = \frac{\mu_0}{4\pi} \frac{i \, ds \times \hat{r}}{r^2} \]  

(Biot–Savart law).

\[ \mu_0 = 4\pi \times 10^{-7} \, \text{T} \cdot \text{m/A} \approx 1.26 \times 10^{-6} \, \text{T} \cdot \text{m/A}. \]

\[ \vec{B} = \int d\vec{B} \]
Magnetic Field due to a Long Straight Wire

This element of current creates a magnetic field at $P$, into the page.

\[ dB = \frac{\mu_0 i \sin \theta}{4\pi r^2} \]

\[ B = 2 \int_0^\infty dB = \frac{\mu_0 i}{2\pi} \int_0^\infty \frac{\sin \theta \, ds}{r^2} \]

\[ r = \sqrt{s^2 + R^2} \]

\[ \sin \theta = \sin(\pi - \theta) = \frac{R}{\sqrt{s^2 + R^2}} \]

\[ B = \frac{\mu_0 i}{2\pi R} \left[ \frac{R}{(s^2 + R^2)^{3/2}} \right]_0^\infty = \frac{\mu_0 i}{2\pi R} \]
29.2: Magnetic Field due to a Long Straight Wire:

\[ B = \frac{\mu_0 i}{2\pi R} \] (long straight wire).

DEMO: needle and wire
DEMO: iron filings on transparency
Magnetic Field due to a Long Straight Wire

The thumb is in the current's direction. The fingers reveal the field vector's direction, which is tangent to a circle.
i-clicker:

Where on the x-axis (excluding infinity) is $B_{tot} = 0$?

a) At a point $x < 0$

b) At a point $x$ such that $0 < x < d$

c) At a point $x > d$

d) Only at infinity
i-clicker:

Where on the x-axis (excluding infinity) is $B_{\text{tot}}=0$?

a) At a point $x < 0$

b) At a point $x$ such that $0 < x < d$

c) At a point $x > d$

d) Only at infinity
At points on wire b, what direction does the magnetic field due to current $i_a$ in “wire a” point?

a) To the right  
b) To the left  
c) Up  
d) Down  
e) it’s zero
i-clicker:

At points on wire b, what direction does the magnetic field due to current $i_a$ in “wire a” point?

a) To the right
b) To the left
c) Up
d) Down
e) it’s zero
i-clicker:

What is the direction of the magnetic force on wire b due to the current in wire a?

a) To the right
b) To the left
c) Up
d) Down
e) it’s zero
i-clicker:

What is the direction of the magnetic force on wire b due to the current in wire a?

a) To the right
b) To the left
c) Up
d) Down
e) it’s zero
Force Between Two Parallel Wires

\[ B_a = \frac{\mu_0 i_a}{2\pi d}. \]

\[ \vec{F}_{ba} = i_b \vec{L} \times \vec{B}_a, \]

\[ F_{ba} = i_b L B_a \sin 90^\circ = \frac{\mu_0 L i_a i_b}{2\pi d}. \]

Parallel currents attract each other, and antiparallel currents repel each other.
Magnetic Field due to a Current in a Circular Arc of Wire:

\[ dB = \frac{\mu_0}{4\pi} \frac{i\,ds\,\sin 90°}{R^2} = \frac{\mu_0}{4\pi} \frac{i\,ds}{R^2}. \]

\[ B = \int dB = \frac{\mu_0 i}{R^2} \int ds = \frac{\mu_0 i}{R^2} R\phi \]

\[ B = \frac{\mu_0 i\phi}{4\pi R} \quad \text{(at center of circular arc).} \]

\[ B = \frac{\mu_0 i(2\pi)}{4\pi R} = \frac{\mu_0 i}{2R} \quad \text{(at center of full circle).} \]

The right-hand rule reveals the field's direction at the center.
Ex:

Find \( B \) at common center of two concentric semicircular loops, radii \( R_1 \) and \( R_2 \).