1. Overview of the relations among charge, field and potential
   - Gauss law
   - Integrate charge to get potential
   - More about energy
   - Laplace and Poisson equations
2. Intro to conductors
   - Field inside is zero
BEFORE CLASS:
Play with metal “cages” – what is the electric field inside?

DO WARM-UP PROBLEMS BEFORE RECITATIONS THIS WEEK
3nd HOMEWORK ASSIGNMENT IS DUE IN CLASS NEXT MONDAY
graded HW1 is here for pickup

class web site
http://www.physics.rutgers.edu/ugrad/272
Charges and Electric Field

Fixed charge distribution
Coulomb’s law
Sum or integrate over the charges to get $\vec{E}(\vec{r})$
Charge distribution determines the electric field
Charges and Electric Field

Fixed charge distribution
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Last time, we “proved” that for any closed surface

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enc}}}{\varepsilon_0}$$  \text{GAUSS’ LAW}

If we know something about the field (usually from symmetry) we can get the field by computing the flux through a surface that includes the point of interest and equating to the charge enclosed
Charges and Electric Field

\[ \oint \mathbf{E} \cdot d\mathbf{A} = \frac{q_{\text{enc}}}{\varepsilon_0} \]  
**GAUSS’ LAW**

\[ \int \nabla \cdot \mathbf{E} \, dV = \int \frac{\rho}{\varepsilon_0} \, dV \]  
true for any closed surface

\[ \nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0} \]  
Gauss law in differential form

Electric field determines the charge distribution
Electric Field and Potential

Given electric field $\vec{E}(\vec{r})$, get potential $\phi(\vec{r})$ by choosing a (simple) path from reference point to point of interest and evaluating a line integral

$$\phi(\vec{r}) = -\int \vec{E} \cdot d\vec{l}$$

Given potential $\phi(\vec{r})$, get electric field $\vec{E}(\vec{r})$ by differentiating

$$\vec{E}(\vec{r}) = -\nabla \phi(\vec{r})$$
Charges and Potential

Fixed charge distribution
Get field from charge distribution
Get potential from field

Seems rather tedious…
Charges and Potential

Fixed charge distribution
Potential of a point charge
Sum or integrate over the charges to get $\varphi(\vec{r})$
Charge distribution determines the potential

Example: charged ring (potential on axis)
Then can get the field from the potential
Energy to assemble a system of charges

Point charges
Continuous charge distribution?

\[ U = \frac{1}{2} \sum_{j=1}^{N} q_j \sum_{k \neq j} \frac{1}{4\pi \varepsilon_0 r_{jk}} q_k \rightarrow \frac{1}{2} \int (\rho(\vec{r}) \, dv) \varphi(\vec{r}) \]

Example: energy to assemble a spherical shell

\[ U = \frac{Q^2}{8\pi \varepsilon_0 R} \]
Energy to assemble a system of charges

\[ U = \frac{1}{2} \sum_{j=1}^{N} q_j \sum_{k \neq j} \frac{1}{4\pi\varepsilon_0} \frac{q_k}{r_{jk}} \rightarrow \frac{1}{2} \int \rho \varphi \, dv \]

Alternative expression: consider that the energy is stored in the electric field
Energy/volume = \( \varepsilon_0 E^2 / 2 \)

For spherical shell \( U = \frac{Q^2}{8\pi\varepsilon_0 R} \)
Apply to systems including one or more point charges. Energy of the electric field of a single point charge is infinite??!

We are really looking at energy DIFFERENCE (energy to bring existing point charge in from infinity) subtract off the infinite self-energy of the point charges in the system

(often need to make sense out of infinity-infinity)
Potential and electric field of a dipole

\begin{align*}
\phi(r, \theta) &= \frac{p \cos \theta}{4 \pi \epsilon_0 r^2} \\
E(r, \theta) &= \frac{p}{4 \pi \epsilon_0 r^3} (2 \cos \theta \hat{r} + \sin \theta \hat{\theta})
\end{align*}

Discussion of the field far away from a generic charge distribution
(do in 1D to reduce the math)
Charges and Potential

Given the potential $\varphi(\vec{r})$
get the field from the potential
get the charge from the field

Combine to get the charge directly from $\varphi(\vec{r})$

$$\nabla \cdot E = -\nabla \cdot \nabla \varphi = \frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} + \frac{\partial^2 \varphi}{\partial x^2}$$

Poisson equation (from Gauss’ law)

$$\nabla^2 \varphi = -\frac{\rho}{\varepsilon_0}$$
Charges and Potential

Another way to get $\varphi(\vec{r})$ from $\rho(\vec{r})$

Poisson equation

Laplace equation in charge-free regions

$\nabla^2 \phi = 0$

$2^{nd}$ order diff equation

“initial conditions” -> “boundary conditions”
Charges and Potential

\[ \phi(\vec{r}) \] solution of Laplace equation \( \nabla^2 \phi = 0 \)

A couple of true statements about the potential in a region with zero charge

See p. 87 of Purcell
Systems with conductors
DEMO: an intriguing observation

Hollow conductor
What is the electric field inside?
DEMO: an intriguing observation

Hollow conductor

Electric field inside the cavity is ZERO
Conductors: (metal, tap water, the human body)

Some fraction of the electrons are free to move anywhere in the material.

*added charge is free to move*

added charges repel each other
will distribute themselves over the surface
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\[ Q_A \quad Q_B \quad Q_A + Q_B \]
Conductors: (metal, tap water, the human body)

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added charge is free to move

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will distribute themselves over the surface

\[ \frac{Q_A + Q_B}{2} \]

\[ \frac{Q_A + Q_B}{2} \]
Three identical spheres are given charges 3 C, 4 C and -1 C. They are brought together into an equilateral triangle, as shown, and then separated again. What are the final charges on the spheres?

(a) Same as the initial charges (3 C, 4 C, -1 C)
(b) All have 1C
(c) All have 2C
(d) All have charge zero.
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will distribute themselves over the surface
Reminder: conductors – charges are free to move through material

Electrostatics:
No electric field in the interior of the material
Otherwise charges would feel an electric force and move

Isolated spherical conductor with excess charge Q
How to arrange the charge to get zero field in the interior?
Reminder: conductors – charges are free to move through material

Electrostatics:
No electric field in the interior of the material
Otherwise charges would feel an electric force and move

Isolated spherical conductor with excess charge $Q$
Charge distributes uniformly on the outside surface
Shell theorem – field inside is zero

Field outside $= kQ/r^2 \ r$
$\sigma = Q/(4 \ \pi \ R^2)$
Magnitude of field at $r = R$ is $k (4 \ \pi \ R^2 \ \sigma)/R^2 = \sigma / \epsilon_0$
Reminder: conductors – charges are free to move through material

Electrostatics:
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Isolated spherical conductor with excess charge Q
Charge distributes uniformly on the outside surface
Shell theorem – field inside is zero

Add a concentric spherical cavity
Reminder: conductors – charges are free to move through material

Electrostatics:
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Shell theorem – field inside is zero

Add a concentric spherical cavity
Electric field is zero inside
DEMO: an intriguing observation

Hollow conductor

Electric field inside the cavity is ZERO

easy for spherical conductor with concentric spherical cavity

(1) Charge distributes itself to outside
(2) Uniform spherical shell
(3) Field inside uniform spherical shell is zero

True for any shape!