TODAY

Review of electromagnetic waves so far
EM waves transport energy
EM waves transport momentum
Polarization of EM waves and cool demos
MAXWELL’S EQUATIONS
FOR TIME-DEPENDENT CHARGE AND CURRENT DISTRIBUTIONS

\[ \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \nabla \times \vec{B} = \mu_0 (\vec{J} + \varepsilon_0 \frac{\partial \vec{E}}{\partial t}) \]

\[ \nabla \cdot \vec{E} = \rho / \varepsilon_0 \quad \nabla \cdot \vec{B} = 0 \]

\( \rho \) and \( dB/dt \) determine \( E \)

J and \( dE/dt \) determine \( B \)

\[ d\vec{B} = \frac{\mu_0}{4\pi} \frac{i d\vec{s} \times \hat{r}}{r^2} \]
Combine the Maxwell equations with \( \rho = j = 0 \) (empty space) to show

\[
\frac{d^2 \vec{E}}{dx^2} + \frac{d^2 \vec{E}}{dy^2} + \frac{d^2 \vec{E}}{dz^2} - \mu_0 \epsilon_0 \frac{d^2 \vec{E}}{dt^2} = 0
\]

\[
\frac{d^2 \vec{B}}{dx^2} + \frac{d^2 \vec{B}}{dy^2} + \frac{d^2 \vec{B}}{dz^2} - \mu_0 \epsilon_0 \frac{d^2 \vec{B}}{dt^2} = 0
\]

\[\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}\]

WAVE EQUATION
Wave solutions for \( E(r,t) \) and \( B(r,t) \)
Wave Motion:

A wave is a disturbance that transports energy away from its source.

“Mechanical waves” propagate in a medium (a string, air, a solid…)
Wave may propagate over a large distance.
But particles of the medium move in a much more localized area.

https://phet.colorado.edu/sims/html/wave-on-a-string/latest/wave-on-a-string_en.html
Combine the Maxwell equations with $\rho = j = 0$ (empty space) to show

$$\frac{d^2 \vec{E}}{dx^2} + \frac{d^2 \vec{E}}{dy^2} + \frac{d^2 \vec{E}}{dz^2} - \mu_0 \varepsilon_0 \frac{d^2 \vec{E}}{dt^2} = 0 \quad \frac{d^2 \vec{B}}{dx^2} + \frac{d^2 \vec{B}}{dy^2} + \frac{d^2 \vec{B}}{dz^2} - \mu_0 \varepsilon_0 \frac{d^2 \vec{B}}{dt^2} = 0$$

compare

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y}{\partial t^2} \quad \text{(wave equation)}.$$  

E and B obey wave equations!

Wave speed = $(\mu_0 \varepsilon_0)^{-1/2}$ = $2.992 \times 10^8$ m/s

Compare measured speed of light
(in 1850, using mirrors 8 km apart)
$c = 3.0 \times 10^8$ m/s

Maxwell’s aha moment!
Simplest example of fields that solve Maxwell’s equations: “Linearly polarized plane wave”

\[ E_y(x, y, z, t) = E_m \sin(kx - \omega t) \]

\[ B_z(x, y, z, t) = B_m \sin(kx - \omega t) \]

\[ \omega / k = c \]

\( E_m \) and \( B_m \) are related by Faraday’s law:
\[ E_m = c B_m \]
Electromagnetic wave traveling in +x direction

E and B fields are perpendicular to direction of travel
E and B are perpendicular to each other
E/B = c

Changing B produces E
Changing E produces B
self-sustaining
Electromagnetic spectrum: Wavelength $\lambda = \frac{c}{f}$
Simple “linearly polarized” electromagnetic wave

Direction of propagation
Wavelength (or frequency)
Direction of the oscillating E field = “polarization”

\[ E_y(x, y, z, t) = E_m \sin(kx - \omega t) \]
\[ B_z(x, y, z, t) = B_m \sin(kx - \omega t) \]
Electromagnetic waves transport energy (radiation from the sun) measure using “intensity” = (energy/area)/time

where $P_s$ is the source power
EM wave transports energy

Energy density associated with E field = \((\varepsilon_0 / 2) |\vec{E}(\vec{r})|^2\)
Energy density associated with B field = \((1 / 2\mu_0) |\vec{B}(\vec{r})|^2\)

\(E_y(x, y, z, t) = E_m \sin(kx - \omega t)\)
\(B_z(x, y, z, t) = B_m \sin(kx - \omega t)\)

Energy/area in one wavelength =
\(\lambda((\varepsilon_0 E_m^2 / 4) + (B_m^2 / (2\mu_0)))\)
\(= \lambda(\varepsilon_0 E_m^2 / 2)\)

(energy associated with E field = energy associated with B field)

Rate at which energy passes through unit area perpendicular to direction of propagation is (energy/area)/time
\(c\varepsilon_0 E_m^2 / 2 = c\varepsilon_0 E_{rms}^2\)
Rate at which energy passes through unit area perpendicular to direction of propagation is

\[ c \varepsilon_0 E_m^2 / 2 = c \varepsilon_0 E_{rms}^2 = I \]

“Energy current density” for a traveling wave

\[ \nabla \cdot \vec{S} = \frac{-\partial U(\vec{r}, t)}{\partial t} \]

\[ \vec{S}(\vec{r}, t) = (\vec{E} \times \vec{B}) / \mu_0 \]

Confirm it gives the right rate for a traveling wave
33.5: Energy Transport and the Poynting Vector:

The direction of the Poynting vector \( \mathbf{S} \) of an electromagnetic wave at any point gives the wave’s direction of travel and the direction of energy transport at that point.

\[
\mathbf{S} = \frac{1}{\mu_0} \mathbf{E} \times \mathbf{B} \quad \text{(Poynting vector)},
\]

Linearly polarized wave \( E \) is perpendicular to \( B \) and to direction of propagation

\[
I = S_{\text{avg}} = \left( \frac{\text{energy/time}}{\text{area}} \right)_{\text{avg}} = \left( \frac{\text{power}}{\text{area}} \right)_{\text{avg}} = \frac{1}{c \mu_0} [E^2]_{\text{avg}} = \frac{1}{c \mu_0} [E_m^2 \sin^2(kx - \omega t)]_{\text{avg}}.
\]

\[
E_{\text{rms}} = \frac{E_m}{\sqrt{2}}.
\]

\[
I = \frac{1}{c \mu_0} E_{\text{rms}}^2.
\]

\[
= c \varepsilon_0 E_m^2 = \text{speed x energy per volume}.
\]
Rate at which energy passes through unit area perpendicular to direction of propagation is

\[ c\varepsilon_0 E_m^2 / 2 = c\varepsilon_0 E_{rms}^2 \]

“Energy current density” for a traveling wave
Direction of the current density = direction of propagation

\[ \nabla \cdot \vec{S} = -\frac{\partial U(\vec{r},t)}{\partial t} \]

\[ \vec{S}(\vec{r},t) = (\vec{E} \times \vec{B}) / \mu_0 \]

Confirm it gives the right rate for a traveling wave
Confirm in general by using vector identities
EM wave also transports **momentum** (direction of momentum is parallel to direction of propagation)

Rate at which this momentum passes through unit area perpendicular to direction of propagation is $I/c$

\[
\text{(momentum/time)/area = force/area = pressure}
\]

“radiation pressure” $P = I/c$

If wave hits a perfectly reflecting surface, then $P = 2I/c$
Simple “linearly polarized” electromagnetic wave

Direction of propagation
Wavelength (or frequency)
Amplitude of the oscillating E field: intensity = power/area
Direction of the oscillating E field = “polarization”

\[ E_y(x, y, z, t) = E_m \sin(kx - \omega t) \]
\[ B_z(x, y, z, t) = B_m \sin(kx - \omega t) \]
An electric field component parallel to the polarizing direction is passed \((transmitted)\) by a polarizing sheet; a component perpendicular to it is absorbed.
An electric field component parallel to the polarizing direction is passed (*transmitted*) by a polarizing sheet; a component perpendicular to it is absorbed.

Component along polarizing direction is transmitted: $E \cos \phi$

Effect is to rotate the polarization and reduce the intensity $I$ proportional to $E_m^2$

So transmitted intensity $I = I_0(\cos \phi)^2$
If the angle is 45 degrees, the intensity decreases by a factor of 2
“Unpolarized” light

Random mixture of directions

e.g. light bulb

One polarizing sheet:
Unpolarized light of intensity $I \rightarrow$ linearly polarized; $I = \frac{I_0}{2}$

Stack another polarizing sheet on top:
Rotate: no difference ---- completely black
Two polarizing sheets are stacked with directions at right angles and placed on the projector, and transmit no light. A third sheet with direction at 45 degrees to each is inserted between the two sheets. What is then true about the intensity $I$ of the light transmitted by the 3-sheet stack?

a) No light is transmitted  
b) $I = I_0$  
c) $I = I_0/2$  
d) $I = I_0/4$  
e) $I = I_0/8$

ANSWER WITH THE DEMO!