1. Force on a charge due to a given charge distribution – setting up integrals
2. Electric field and electric field lines
3. Electric flux (surface integrals)
4. Gauss’ law
BEFORE CLASS:
A BIG CHARGED METAL SPHERE
what happens to nearby objects? To objects that
touch the sphere? Let’s find out!

FIRST HOMEWORK ASSIGNMENT IS DUE IN
CLASS TODAY
2nd HOMEWORK ASSIGNMENT IS DUE IN
CLASS NEXT MONDAY

class web site
http://www.physics.rutgers.edu/ugrad/272
Force on a charge due to a given charge distribution

Collection of point charges – vector sum

Linear charge distributions
- Straight wire
- Circular arc / ring

Divide in tiny pieces
Find \(dQ\) (recitation last week)
Find contribution of \(dQ\) to the force
Sum -> integral
Thin wire of length L has total charge Q distributed uniformly. Find the electric force on point charge q at any point \( x_q > L \).

Define a linear charge density: \( \lambda = \frac{Q}{L} \)

\[
dQ = \lambda \, dx
\]

\[
dF = \frac{k \, q \, \lambda \, ds}{(x_q-x)^2} \quad \text{integrate from} \quad x_L \quad \text{to} \quad x_R
\]

\[
F = \frac{k \, q \, Q}{((x_q-x_L)(x_q-x_R))}
\]

How to take the limit as \( x \) gets far away? If \( x_m \) is the midpoint,

\[
x_L = x_m - \frac{L}{2}, \quad x_R = x_m + \frac{L}{2}
\]

\[
F = \frac{k \, q \, Q}{((x_q-x_m+L/2)(x_q-x_m-L/2))} = \frac{k \, q \, Q}{((x_q-x_m)^2-L^2/4)}
\]

\[
F = \left( \frac{k \, q \, Q}{(x_q-x_m)^2} \right)(1-(L/(x_q-x_m))^2/4)
\]

For \( L/(x_q-x_m) \ll 1 \), this goes to \( F = \frac{k \, q \, Q}{(x_q-x_m)^2} \)

looks like point charge Q at \( x_m \)
Rod of charge $Q$ is bent into semicircle of radius $R$.
Linear charge density $\lambda = Q/\pi R$
Find electric force on charge $q$ at center.

Parametrize the curve by angle $\theta$
$dQ = \lambda R \, d\theta$

Vertical component vanishes by symmetry

Horizontal component
Integrate $(\sin \theta) \, dF$ from $\theta = 0$ to $\theta = \pi$

$$E = 2 \, k \frac{\lambda}{R} \text{ to the right}$$
Electric field of uniformly charged ring on its central axis

\[ F = k \frac{q Q z}{(z^2 + R^2)^{3/2}} \]

For \( z \gg R \), looks like point charge \( Q \)
Electric field
\[ \vec{F} = q_1 \left( \frac{kq}{r^2} \right)^\hat{r} \]

\[ \vec{r} = \text{vector from } q \text{ to } q_1 \]

\[ \hat{r} = \vec{r} / |\vec{r}| \]
\[ \vec{F} = q_2 \left( \frac{kq}{r^2} \right) \hat{r} \]

\[ \vec{E}(\vec{r}) = \left( \frac{kq}{r^2} \right) \hat{r} \]

\[ \vec{F} = q_2 \vec{E} (\vec{r}) \]
Electric field is force /charge, units of N/C

Charge q modifies the space around it
Not just a mathematical construct, but a physical reality
Electric field lines point in the direction of $\vec{E}$ at any point.

Spacing of lines decreases as magnitude of $\vec{E}$ increases.

(b) $q>0$

$\quad q<0$
Electric field outside a uniformly charged spherical shell
Electric field of a Van de Graff generator

The graphite spheres on a stick
A single charge on a line

Q, \( x_0 \)
A single charge on a line

\[ E = \frac{-kQ}{(x-x_0)^2} \quad Q, \ x_0 \quad E = \frac{kQ}{(x-x_0)^2} \]

E>0 field points to the right
E<0 field points to the left
Two charges on a line

$Q_1, x_1$ $Q_2, x_2$
Two charges on a line

\[ E(x) = -\frac{kQ_1}{(x-x_1)^2} - \frac{kQ_2}{(x-x_2)^2} \text{ left} \]
\[ E(x) = +\frac{kQ_1}{(x-x_1)^2} - \frac{kQ_2}{(x-x_2)^2} \text{ middle} \]
\[ E(x) = +\frac{kQ_1}{(x-x_1)^2} + \frac{kQ_2}{(x-x_2)^2} \text{ right} \]
Two charges on a line

If $Q_2 = 4Q_1$ and $x_2 - x_1 = d$, where is the E field equal to 0?
The figure shows four systems in which charges are arranged on a line. In which system is the magnitude of the electric field at the central point GREATEST?
The figure shows four systems in which charges are arranged on a line. In which system is the magnitude of the electric field at the central point GREATEST?
Definition of dipole

Electric field of the dipole?
Electric field at points on the x-axis
Electric field at points on the x-axis
Electric field at points on the y-axis
Electric field of continuous charge distributions

Same integrals as force at beginning of lecture

\[ E = \frac{F}{q} \]
ELECTRIC FLUX

2D surfaces in 3D space
Flux through a flat surface (area A, unit vector n)
Uniform E field

Nonzero flux example
Zero flux example
Open vs closed surfaces

Closed surface: unit vector $n$ points from inside to outside
CHECKPOINT 1

The figure here shows a Gaussian cube of face area $A$ immersed in a uniform electric field $\vec{E}$ that has the positive direction of the $z$ axis. In terms of $E$ and $A$, what is the flux through (a) the front face (which is in the $xy$ plane), (b) the rear face, (c) the top face, and (d) the whole cube?

\[ \vec{E} = E \hat{z} \]

Front surface
\[ \hat{n} = \hat{z} \]
\[ \vec{E} \cdot \hat{n} = E \]
Multiply by $A$ to get $EA$

Side surfaces
\[ \vec{E} \cdot \hat{n} = 0 \]

Back surface
\[ \hat{n} = -\hat{z} \]
contribution is $-EA$

Flux integral is $EA - EA = 0$
Graphical approach:

Flux integral is proportional to 
# of field lines out - # of field lines in

Useful when you need to decide if the flux is zero, positive or negative
Flux through a non flat surface
Nonuniform E field

Divide into tiny pieces – close to flat and close to uniform
Add up contribution of tiny pieces – surface integral

For now we will confine our attention to relatively simple
surfaces (spheres, cylinders)
and simple nonuniform fields (flux through each tiny piece
of the surface is the same)
A single point charge $q$: $E$ is given by Coulomb’s law

Consider spherical surface $S$ of radius $R$ centered at point charge
Flux integral of $\mathbf{F}(\mathbf{r})$ over $S$

Divide the surface into tiny pieces

Find the normal vector to the piece
Compute the real-number-valued function $\mathbf{F}(\mathbf{r}) \cdot \mathbf{n}$

For each tiny piece, multiply the area of the piece by this number
Sum up the contributions from all the pieces

Surface integral of the function $\mathbf{F}(\mathbf{r}) \cdot \mathbf{n} = \text{flux integral}$
GAUSS LAW
A single point charge \( q \): \( \mathbf{E}(x,y,z) = \frac{kq \mathbf{r}}{r^2} \)

Consider a spherical surface \( S \) of radius \( R \) centered at point charge.

Flux integral

\[ \oint \mathbf{E} \cdot d\mathbf{A} \]

\( \mathbf{F}(\mathbf{r}) \) dot \( \mathbf{n} \)-hat

\( \mathbf{n} \)-hat = \( \mathbf{r} \)-hat

\( \frac{kq \mathbf{r}}{R^2} \) dot nhat = \( \frac{kq}{R^2} \)

Constant on the surface

Surface integral is \( 4\pi R^2 \frac{kq}{R^2} \)

= flux integral = \( 4\pi kq = \frac{q}{\varepsilon_0} \)

THE SAME FOR ANY \( R \)!

\[ \oint \mathbf{E} \cdot d\mathbf{A} = q_{\text{enc}}/\varepsilon_0 \]
This expression is true for a single point charge and spherical surfaces centered on the point

Other charge distributions? Eg point charge inside but not at center

What about other surfaces?