TODAY:
Can crusher followup
Inductors in circuits
Simple circuits that depend on time
• RC review
• RL
• LC
BEFORE CLASS:
COME AND PLAY WITH DEMOS

EXAM #2 IS WED APRIL 11 IN CLASS

class web site
http://www.physics.rutgers.edu/ugrad/272
pop quiz info

Out of 23 papers

9 (check plus) were fine

7 (check): did not show how to get B
OR wrong sign for cross product in computing force

7: (check minus or minus) showed serious confusion
EXAMPLE: UNIFORM B FIELD with changing magnitude inside solenoid

INDUCED E FIELD

IF THERE IS A WIRE LOOP on the curve C, current will flow
MAGNETIC FORCE ON THE WIRE LOOP
CURRENT PULSE: I, B increase from zero
CAN CRUSHER DEMO
CAN CRUSHER DEMO
CURRENT PULSE: I, B increase from zero (RLC with switch closed at t=0)

Magnetic force on loop is INWARD
CURRENT FLOW IS THE SAME AS IF THERE WERE AN EMF SOURCE INSERTED INTO THE LOOP
Simple time-varying circuits

Let’s first review RC circuits from last time
Switch closed to “a” position at t=0
Initial q = 0

\[ \mathcal{E} - iR - \frac{q}{C} = 0. \]

\[ i = \frac{dq}{dt}. \]

\[ R \frac{dq}{dt} + \frac{q}{C} = \mathcal{E} \]

\[ q = C \mathcal{E} (1 - e^{-t/RC}) \]

\[ i = \frac{dq}{dt} = \left( \frac{\mathcal{E}}{R} \right) e^{-t/RC} \]

**Fig. 27-15** When switch S is closed on a, the capacitor is *charged* through the resistor. When the switch is afterward closed on b, the capacitor *discharges* through the resistor.

A capacitor that is being charged initially acts like ordinary connecting wire relative to the charging current. A long time later, it acts like a broken wire.
RC circuits

When switch $S$ is closed on $a$, the capacitor is charged through the resistor. When the switch is afterward closed on $b$, the capacitor discharges through the resistor.

The time it takes for the current to decrease by a factor of $e$ is the "time constant".

$$\tau = RC$$

$$i = \frac{dq}{dt} = \left(\frac{\mathcal{E}}{R}\right)e^{-t/RC}$$

$RC \ln 2 =$ the time it takes for the current to decrease by a factor of 2
Discharge the capacitor

\[ R \frac{dq}{dt} + \frac{q}{C} = 0 \]

\[ q = q_0 e^{-\frac{t}{RC}} \]

\[ i = \frac{dq}{dt} = -\left( \frac{q_0}{RC} \right) e^{-\frac{t}{RC}} \]

\[ \tau = RC \]

the time it takes for the current to decrease by a factor of e “time constant”

RC ln 2 = the time it takes for the current to decrease by a factor of 2
Discharge the capacitor

\[ R \frac{dq}{dt} + \frac{q}{C} = 0 \]

\[ q = q_0 e^{-\frac{t}{RC}} \]

\[ i = \frac{dq}{dt} = -\left( \frac{q_0}{RC} \right) e^{-\frac{t}{RC}} \]

the time it takes for the current to decrease by a factor of e

“time constant”

RC ln 2 = the time it takes for the current to decrease by a factor of 2

What happened to the energy stored in the capacitor?
i^2R in the resistor – dissipated as heat
A new circuit element: the inductor

$L$
Wire coil with resistance $R=0$ and current $i$

Current $i$ in loop produces a magnetic flux through loop $\Phi = L \, i$

$L$ is called the self-inductance of the loop

Units: $1 \, T \, m^2 / A = 1 \, H$ (1 henry)

If the current changes, the magnetic flux will change $\Phi(t) = L \, i(t)$

Effect is to create electric fields in the loop

These fields induce current that opposes the change in current

Like inserting a battery with $\mathcal{E} = |d\Phi(t)/dt| = L \, di/dt$
A new circuit element: the inductor

You can still apply the loop rule If as you go through the inductor in the direction of the current arrow, you count a voltage difference \( = -L \frac{di(t)}{dt} \)

Note that if \( i \) does not vary with time, there is NO voltage difference
30.9: RL Circuits:

Switch closed to “a” position at $t=0$

$$-iR - L \frac{di}{dt} + \mathcal{E} = 0$$

$$i = \frac{\mathcal{E}}{R} \left(1 - e^{-Rt/L}\right),$$

$\tau_L = \frac{L}{R}$ (time constant).

Initially, an inductor acts to oppose changes in the current through it. A long time later, it acts like ordinary connecting wire.
At time $t = (L/R)\ln 2$, what is the voltage across the inductor?

(a) $\frac{E}{2}$

(b) $E \ln 2$

(d) $\frac{E}{(\ln 2)}$

(e) I have no idea

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**Fig. 30-15** An $RL$ circuit. When switch $S$ is closed on $a$, the current rises and approaches a limiting value $\frac{E}{R}$. 
At time $t = (L/R) \ln 2$, what is the voltage across the inductor?

(a) $E/2$
(b) $E \ln 2$
(d) $E/(\ln 2)$
(e) I have no idea

Fig. 30-15  An $RL$ circuit. When switch $S$ is closed on $a$, the current rises and approaches a limiting value $E/R$. 
LC circuits

\[ \frac{Q(t)}{C} - L \frac{di(t)}{dt} = 0 \]
\[ i(t) = -\frac{dQ(t)}{dt} \]
\[ \frac{Q(t)}{C} + L \frac{d^2Q(t)}{dt^2} = 0 \]

\[ Q(t=0) = q_0 \]
\[ i(t=0) = 0 \]

\[ Q(t) = q_0 \cos \omega t \]

where \( \omega = \frac{1}{(LC)^{1/2}} \)
LC circuits

\[ Q(t) = q_0 \cos \omega t \]

where \( \omega = \frac{1}{(LC)^{1/2}} \)

\[ i(t) = \omega q_0 \sin \omega t \]
30.10: Energy Stored in a Magnetic Field:

To increase the current from zero, the battery has to do work against the emf induced by the change in the current.

\[ \mathcal{E} = L \frac{di}{dt} + iR, \]

\[ \frac{dU_B}{dt} = Li \frac{di}{dt}. \]

\[ \mathcal{E}i = Li \frac{di}{dt} + i^2R, \]

\[ \int_0^U_B dU_B = \int_0^i Li \, di \]

\[ U_B = \frac{1}{2} Li^2 \quad \text{(magnetic energy)}, \]
LC circuits

\[ Q(t) = q_0 \cos \omega t \]
where \( \omega = 1/(LC)^{1/2} \)

\[ i(t) = \omega q_0 \sin \omega t \]

Energy \( Q^2/(2C) + Li^2/2 \) is conserved
Alternates between all in the capacitor and all in the inductor