4.21(a) Current pulse - plane electrodes

\[ I = \frac{-dQ_2}{dt} \quad \text{where} \quad Q_2 \text{ is the total charge on the surface of the right-hand plate} \]

We are told \( Q_2 = -\frac{Qx}{S} \)

\[ I = \frac{Q}{S} \frac{dx}{dt} = \frac{QV}{S} = \frac{2 \times 1.6 \times 10^{-19} \text{ C} \times 10^6 \text{ m}}{2 \times 10^{-3} \text{ m} / \text{s}} \]

\[ I = 1.6 \times 10^{-10} \text{ A} \]

I drops to zero when the particle arrives at the right-hand plate.

\[ t = \frac{S}{V} = \frac{2 \times 10^{-3} \text{ m}}{10^6 \text{ m/s}} = 2 \times 10^{-9} \text{ s} \]

If the particle moves at an angle of 45°, then \[ \frac{dx}{dt} = \frac{1}{12} \times 10^6 \text{ m/s} \]

I is decreased by a factor of \( \sqrt{2} \), travel time is increased by \( \sqrt{2} \).
#2 (4.38 Two light bulbs)

(a) \[ P_1 = 2P_2 \quad (P = \text{power dissipated}) \]

Since \[ P = \frac{V^2}{R} \]

\[ V^2 = \frac{2V^2}{R_2} \Rightarrow R_2 = 2R_1 \]

(b) \[ \text{Since } P = I^2R \]

\[ P_2^{(s)} = I^2R_2 = I^22R_1 = 2P_1^{(s)} \]

(superscript (s) stands for series)

Relate to parallel configuration in (a)

\[ I (R_1 + R_2) = V \Rightarrow I = \frac{V}{(3R_1)} \]

\[ P_1^{(s)} = I^2R_1 = \frac{V^2}{(3R_1)^2} \cdot \frac{1}{R_1} = \frac{1}{9} \cdot \frac{V^2}{R_1} = \frac{1}{9}P_1 \]

\[ P_2^{(s)} = I^2R_2 = \frac{V^2}{(3R_1)^2} \cdot 2R_1 = \frac{2}{9} \cdot \frac{V^2}{R_1} = \frac{2}{9}P_1 \]
#3 (4.39 Maximum power)

\[
\begin{align*}
I &= \frac{\mathcal{E}}{R_1 + R} \\
P &= I^2 R = \frac{\mathcal{E}^2 R}{(R_1 + R)^2} \\
\text{maximize } P \text{ as a function of } R \\
\frac{dP}{dR} &= \frac{\mathcal{E}^2}{(R_1 + R)^2} - \frac{2 \mathcal{E}^2 R}{(R_1 + R)^3} = 0 \\
\mathcal{E}^2 (R_1 + R) &= 2 \mathcal{E}^2 R_m \Rightarrow R_m = R_1 \\
\text{check this is a maximum:} \\
\left.\frac{d^2P}{dR^2}\right|_{R=R_1} &= -\frac{2 \mathcal{E}^2}{(R_1 + R)^3} - \frac{2 \mathcal{E}^2}{(R_1 + R)^3} + \frac{6 \mathcal{E}^2 R}{(R_1 + R)^4} \\
&= \frac{\mathcal{E}^2}{(2R_1)^3} \left(-4 + \frac{6R_1}{2R_1}\right) < 0 \Rightarrow \text{maximum}. 
\end{align*}
\]
#4 (4.40 Minimum power dissipation)

\[ I_1 + I_2 = I_0 \]

\[ P = I_1^2 R_1 + (I_0 - I_1)^2 R_2 \]

**minimize** \( P \) wrt \( I_1 \)

\[ \frac{dP}{dI_1} = 2I_1 R_1 - 2(I_0 - I_1)R_2 = 0 \]

\[ \frac{d^2P}{dI_1^2} = 2R_1 + 2R_2 > 0 \]

\[ I_1(R_1 + R_2) = I_0 R_2 \]

\[ I_1 = \frac{R_2 I_0}{R_1 + R_2}, \quad I_2 = \frac{R_1 I_0}{R_1 + R_2} \]

**using ordinary circuit formulas**

\[ I_1 R_1 = I_2 R_2, \quad I_1 + I_2 = I_0 \]

so

\[ I_1 + \frac{I_1 R_1}{R_2} = I_0 \Rightarrow I_1 = \frac{R_2 I_0}{R_1 + R_2}, \quad I_2 = I_0 - I_1 = \frac{R_1 I_0}{R_1 + R_2} \]

the same as above.
Keeping the same resistance

\[ R_{eq} = R_1 + \left( \frac{1}{R_1} + \frac{1}{R_1 + R_0} \right)^{-1} = R_0 \]

\[ R_1 + \frac{R_1 (R_1 + R_0)}{2R_1 + R_0} = R_0 \]

\[ 2R_1^2 + R_1R_0 + R_1^2 + R_1R_0 = 2R_0R_1 + R_0^2 \]

\[ R_0^2 = 3R_1^2 \quad \Rightarrow \quad \boxed{R_1 = \frac{R_0}{\sqrt{3}}} \]
4.35 Resistances in a cube

(a) 
Circle 0 are at some potential and circle 0 are at same potential collapse to one point

\[ \frac{R}{3} + \frac{R}{6} + \frac{R}{3} = \frac{5R}{6} \]

(b) 
By symmetry, vertices marked with same symbol are at same potential

Each dot should have outer 3 or 6 resistors coming in, total of 12

Combine resistors in parallel and in series

\[ \frac{R}{2} \quad \frac{R}{2} \]

\[ 3R/2 \quad 3R/2 \]

Analyze this circuit to get effective resistance
apply loop rules:

\[ E - \frac{i_1 R}{2} - \frac{(i_1-i_3) R}{2} = 0 \]

\[ -\frac{i_1 R}{2} - \frac{i_3 R}{2} + \frac{i_2 3R}{2} = 0 \]

\[ -(i_2+i_3) \frac{3R}{2} + (i_1-i_3) \frac{R}{2} - i_3 \frac{R}{2} = 0 \]

\[ \lambda_3 = 0 \quad i_1 = \frac{E}{R} \quad i_2 = \frac{E}{3R} \]

\[ R_{\text{eff}} = \frac{E}{\lambda_1+i_2} \frac{4 \frac{E}{3R}}{4} = 3R \]
(c) adjacent corners

A \[ \rightarrow \] B

Reduce using series & parallel:

\[ \text{Reff in central link} \]
\[ \left( \frac{1}{2R} + \frac{2}{R} \right)^{-1} = \frac{2R}{5} \]

\[ \text{Reff} = \left( \frac{5 + \frac{1}{7R}}{7R} \right)^{-1} = \frac{7R}{12} \]
4.3.4 Effective resistances in lattices

Repeat the steps for the 2D square lattice to find the current flowing between adjacent nodes as a fraction of the total current flowing in one node and out the other.

a) 3D cubic lattice: \( i \rightarrow \frac{i}{6} \) (6 segments per node)
   \[ R_{eq} = 2 \cdot \frac{i}{6} \cdot R = \frac{R}{3} = \frac{1}{3} \Omega \]

b) 2D triangular lattice: 6 segments per node
   \[ R_{eq} = \frac{R}{3} = \frac{1}{3} \Omega \]

c) 2D hexagonal "lattice"
   \[ 3 \text{ segments/node} \]
   \[ R_{eq} = 2 \cdot \frac{i}{3} \cdot R = \frac{2R}{3} = \frac{2}{3} \Omega \]

(d) 1D lattice
   \[ 2 \text{ segments/node} \]
   \[ R_{eq} = 2 \cdot \frac{i}{2} \cdot R = R = 1 \Omega \]

("silly" because it is easy to find the current distribution w/o the symmetry supposition — all the current through the adjoining segment satisfies Ohm's law for the whole lattice.)
4.36 Attenuator chain

Let the equivalent resistance of the infinite attenuator chain be \( R \).

Adding one more step to the infinite chain cannot change its resistance, so the resistance of the network below is \( R \).

\[
A' \quad R_1 \quad B'
\]

\[
R = R_1 + RR_2 \quad \text{Solve for } R
\]

\[
R_2 \leq \frac{2R}{R + R_2}
\]

\[
R^2 + R_2 R = R_1 R + R_1 R_2 + RR_2
\]

\[
R^2 - R_1 R - R_1 R_2 = 0
\]

\[
R = R_1 \pm \sqrt{R_1^2 + 4R_1 R_2}
\]

Take the + sign for \( R > 0 \)

\[
R = \frac{R_1 + \sqrt{R_1^2 + 4R_1 R_2}}{2}
\]
4.45 Battery/resistor loop

open circuit: single loop

\[ +(1.5V) \cdot 2 + 1.5V - 5i \cdot 100 \Omega = 0 \]
\[ i = \frac{4.5}{500} A = 0.009 A \]

\[ V_B = V_A + 3.0V - 300 \Omega \times 0.009A \]
\[ = V_A + 0.3V \]

short-circuit current

\[ 3V - 3i_1 \cdot 100 \Omega = 0 \]
\[ i_1 = 0.01 A \]

\[ 1.5V - 2i_2 \cdot 100 \Omega = 0 \]
\[ i_2 = 0.0075 A \]

\[ i_{sc} = i_1 - i_2 = 0.0025 A \]

Therein equivalent

\[ \frac{0.3V}{1} \cdot \frac{\text{m} \cdot 100 \Omega}{i} \]

\[ R_{eq} = \frac{0.3 \Omega}{0.0025} = 120 \Omega \]
4.46 Maximum power via Thevenin

\[ 120V \]
\[ 10\Omega \]
\[ \text{open circuit} \]
\[ 120V - 20A = 0 \]
\[ V_B = V_A + 120V - 60V = V_A + 60V \]

Find \( R_{eq} \) by removing emf source and computing equivalent resistance

\[ R_{eq} = 15 + \left( \frac{1}{10} + \frac{1}{10} \right)^{-1} = 20\Omega \]

Thevenin eq: \[ A \quad 20\Omega \quad 60V \quad B \]

Use results of 4.39 - if \( R \) is connected, maximum power is delivered when \( R = R_{eq} = 20\Omega \)