From (a), we know \( \Phi_E \cdot dA = \frac{9}{5} \).

By symmetry, all quadrants of \( A' \) have the same flux, so \( \Phi_E \cdot dA = \frac{9}{5} \).

By symmetry, \( \Phi_B = \Phi_C = \Phi_A = \frac{9}{5} \).

\( \Phi_B = \Phi_C = \Phi_A = \frac{9}{5} \) because the \( E \)-field lines are parallel to these faces, so \( \Phi_E \cdot dA = 0 \).
This assumption is based on symmetry of the cylinder (rotations around the axis, translations along the axis, mirror through a plane perpendicular to the axis, mirror though a plane containing the axis)
Building a sheet from rods

\[ E_x \text{ and } E_y \text{ cancel out due to symmetry (at pt. P)} \]

For a single rod:

\[ |E| = \frac{\lambda}{2\pi \varepsilon_0 r} \quad \text{where} \quad r = \sqrt{x^2 + z^2} \]

Electric field due to Sheet of rods

\[ dE = \frac{d\lambda}{2\pi \varepsilon_0} \left( \frac{1}{\sqrt{x^2 + z^2}} \right) \]

\[ \therefore \quad dE_z = \frac{d\lambda}{2\pi \varepsilon_0} \left( \frac{1}{\sqrt{x^2 + z^2}} \right) \cos \theta = \frac{d\lambda}{2\pi \varepsilon_0} \left( \frac{1}{\sqrt{x^2 + z^2}} \right) \left( \frac{z}{\sqrt{x^2 + z^2}} \right) \]

\[ E_z = \int_{-\infty}^{\infty} \frac{d\lambda \: z}{2\pi \varepsilon_0 (x^2 + z^2)} \]

\[ E_z = \int_{-\infty}^{\infty} \frac{\sigma \: z}{2\pi \varepsilon_0 (x^2 + z^2)} \: dx \]

\[ E_z = \frac{\sigma}{2\pi \varepsilon_0} \left( \frac{1}{2} \right) \tan^{-1} \left( \frac{x}{z} \right) \bigg|_{-\infty}^{\infty} \]

\[ = \frac{\sigma}{2\pi \varepsilon_0} \left( \frac{\pi}{2} - \left( -\frac{\pi}{2} \right) \right) = \frac{\sigma}{2\pi \varepsilon_0} \left( \pi \right) = \frac{\sigma}{2 \varepsilon_0} \]

\[ \therefore \quad |E| = \frac{\sigma}{2 \varepsilon_0} \]
Pt A: $\vec{E}$ due to sphere $b=0$ due to the shell theorem.

$\vec{E}$ due to sphere $a = \frac{Kq}{(g_2)^2}$, $q = -p\left(\frac{4}{3}\pi\left(\frac{a}{2}\right)^3\right)$

(using it like a pt. charge, again due to the shell theorem)

$$\vec{E}_a = \frac{Kp}{(g_2)^2} \cdot \frac{2}{3} \pi \left(\frac{a}{2}\right)^3 = \frac{2Kp\pi a}{3}, \text{pointing up}$$

(the field points towards the 'negative' charge of sphere $a$)

Pt B: Using the Shell Theorem

$$\vec{E}_a = \frac{K}{(g_2)^2} r^3 \left(\frac{4}{3}\pi \left(\frac{a}{2}\right)^3\right) = \frac{K}{\pi (g_2)^2} \left(\frac{2}{3}\pi \left(\frac{a}{2}\right)^3\right) = \frac{2}{27} Kp\pi a$$

$r = a + \frac{g_2}{2}$

(pointing away from center = pointing down = -)

$$\vec{E}_b = \frac{K}{a^2} r^3 \left(\frac{4}{3}\pi \left(\frac{a}{2}\right)^3\right) = -\frac{4}{3} Kp\pi a$$

$$\vec{E} = \left(\frac{2}{27} - \frac{4}{3}\right) Kp\pi a = -\frac{34}{27} Kp\pi a$$

$$\vec{E} = \frac{34}{27} Kp\pi a \text{ pointing down}$$
Sheet: \[ \int E_{\text{sheet}} \cdot dA = \frac{q}{\varepsilon_0} \quad A = \text{Area} \]
- \[ Q = \sigma A \]
- \[ E_{\text{sheet}} = \frac{Q}{2 \varepsilon_0} \]
  (pointing away from sheet for \( \sigma \) positive)

Slab: \[ \int E_{\text{slab}} \cdot dA = \frac{Q}{\varepsilon_0} \]
- \[ Q = \frac{1}{2} \sigma A \]
- \[ E_{\text{slab}} = \frac{Q}{2 \varepsilon_0} \]
  (pointing away from slab for \( \sigma \) positive)

Region 1: \[ E_1 = \frac{E_{\text{sheet}} + E_{\text{slab}}}{2} \]
- \[ E_1 = \frac{Q}{2 \varepsilon_0} + \frac{Q}{2 \varepsilon_0} \quad \text{(to the left)} \]

Region 2: \[ E_2 = \frac{Q}{2 \varepsilon_0} + \frac{Q}{2 \varepsilon_0} - \frac{x^2}{2 \varepsilon_0} \quad x = \text{distance from right side of slab} \]

Region 3: \[ E_3 = \frac{E_{\text{sheet}} + E_{\text{slab}}}{2} \]
- \[ E_3 = \frac{Q}{2 \varepsilon_0} + \frac{Q}{2 \varepsilon_0} \quad \text{(to the right)} \]
Use superposition to find the field.

Use Gauss's law to find the electric field of the cylinder.

\[ \oint E \cdot dA = \frac{q}{\varepsilon_0} \quad q = \sigma \pi r_c^2 h \]

\[ E_{\text{cyl}} \cdot 2\pi r_c \Delta z = \frac{\sigma \pi r_c^2 \Delta z}{\varepsilon_0} \Rightarrow E_{\text{cylinder}} = \frac{\sigma r_c}{2\varepsilon_0} \quad \text{if distance from origin} \]

From \((a,0,0)\):

\[ \oint E \cdot dA = \frac{q}{\varepsilon_0} \quad q = \frac{\sigma}{2} \pi r_s^3 \cdot \frac{-3\rho}{2} \]

\[ E_{\text{sphere}} \cdot 4\pi r_s^2 = -2\pi r_s^3 \rho \Rightarrow E_{\text{sphere}} = -\frac{\rho r_s}{2\varepsilon_0} \hat{r} \]

\[ E_{\text{cylinder}} = \frac{\rho}{2\varepsilon_0} \hat{r} + \frac{\rho}{2\varepsilon_0} \left( x \hat{i} + y \hat{j} \right) \]

\[ E_{\text{sphere}} = -\frac{\rho r_s}{2\varepsilon_0} \hat{r} \]

\[ E_{\text{cylinder}} = \frac{\rho}{2\varepsilon_0} \left( x \hat{i} + y \hat{j} \right) \]

\[ E_{\text{sphere}} = -\frac{\rho}{2\varepsilon_0} \left( (x-a) \hat{i} + (y-y_f) \hat{j} \right) \]

The electric field in the xy-plane of the sphere is simply the superposition of the two fields.

\[ E_{\text{xy-plane}} = \frac{\rho}{2\varepsilon_0} \left( (x-a) \hat{i} + (y-y_f) \hat{j} \right) \]

\[ E_{\text{xy-plane}} = \frac{\rho \cdot a}{2\varepsilon_0} \hat{r} \]

The electric field is constant in the xy-plane within the sphere, always points in the \(\hat{r}\) direction, and has a magnitude of \(\frac{\rho \cdot a}{2\varepsilon_0}\).
Electron jelly:

Forces on proton from each other $\Rightarrow$ equal and opposite.

Forces on proton from jelly $\Rightarrow$ equal and opposite. $\therefore$ They are equidistant from center.

Force on a proton radius $r$ away from center $\propto$ amount of charge inside sphere of radius $r$.

$$q_{\text{enc}} = \frac{r^3}{a^3} (-2q)$$

$\therefore$ Charge inside jelly $= -2q$.

$\vec{F}$ on one of the protons $= 0$

$$\vec{F} = \frac{q^2}{(2\pi r)^2} \hat{r} + \frac{q}{a} \left( \frac{r^3}{a^3} \right) (-2q) \hat{r} = 0$$

$$\Rightarrow \frac{q^2}{4\pi r^2} = 2q^2 \left( \frac{a}{a^3} \right) = 0$$

$$\Rightarrow \frac{q^2}{4\pi r^2} = 2q^2 \left( \frac{a}{a^3} \right) \Rightarrow \frac{a^3}{q} = \pi$$

$\therefore$ $r = \frac{a}{2}$
8. Purcell 1.78: Hole in a shell

Let’s assume that the disk is back on the spherical shell. Therefore, we know that the electric field due to the disk must cancel the electric field due to the rest of the sphere (since $E = 0$ inside a conducting shell of charge).

We also know that the electric field at a point on the plane of a disk is distance $y$ above its center is $\frac{E}{2\pi y}$ if $y$ is much less than $b$. Since we are concerned with the center of the aperture, the $E$ field can indeed be considered $\frac{E}{2\pi a}$.

Because the fields must cancel,

we can conclude that the field of the aperture is of magnitude $\frac{E}{2\pi a}$ and points outwards (parallel with the radius).
9. \( \ddot{a} \): Finding the potential

\[
\begin{array}{c}
\text{(0,0)} \rightarrow \text{(x,0)} \rightarrow \text{(x,y)} \rightarrow \text{(0,y)} \rightarrow \text{(0,0)}
\end{array}
\]

\[E_z = \langle 6xy, 3x^2 - 3y^2, 0 \rangle\]

- When calculating \( \int \vec{E} \cdot ds \) along \( y = c \) or \( x = c \), we only need to consider the \( E_x \) or \( E_y \) respectively, since the dot product will diagonalize the normal component.

**Path 1**

\[
\int_{P_0}^{P_1} \vec{E} \cdot ds - y = 0, \quad E_x = 0 \text{ during this path.}
\]

\[
\int_{P_0}^{P_1} \vec{E} \cdot ds = \int_0^1 3x \, dx = 3x^2 \bigg|_0^1 = 3x^2 y_1 - y_1^3
\]

**Path 2**

\[
\int_{P_0}^{P_2} \vec{E} \cdot ds = \int_0^1 3(0^2 - 3y^2) = -y_1^3
\]

\[
\int_{P_0}^{P_2} \vec{E} \cdot ds = \int_0^x 6y \, dx = 3y_1 \left( x \bigg|_0^1 \right) = 3x^2 y_1
\]

This work is the potential energy, let it be

\[
\bar{E}(x,y) = -(3x^2 y - y^3)
\]

\[
\bar{E} = -\nabla \cdot \bar{E} = -\left( \nabla \cdot (3x^2 y - y^3) \right) = \langle 6xy, 3x^2 - 3y^2, 0 \rangle
\]
2.37) \( R_e = 6.4 \times 10^6 \text{ m} \)

Since we are just outside the shell, we can condense the sphere into a point charge at the center of Earth.

\[
\vec{E} = \frac{kq}{r^2} \quad \quad \bar{E} = \frac{16\pi \varepsilon_0 \cdot 10^9 \frac{N m^2}{C^2}}{(6.4 \times 10^6)^2}
\]

\[
\vec{E} = 2.195 \times 10^{-11} \frac{N}{C}
\]

We can still use the point charge logic when finding the potential (only applies for points outside of the sphere).

**Potential inside sphere:** 0

\[
\phi(r) = \int_r^\infty \frac{kq}{x} \, dx = \left[ -\frac{kq}{x} \right]_r^\infty = 0 + \frac{kq}{r}
\]

\[
\phi(r) = \frac{kq}{r} \quad \text{for } r \geq R_e
\]

\[
\phi(R_e) = \frac{k \cdot 1e}{R_e} = 1.405 \times 10^3 \text{ Volts at } R_e \text{ potential of sphere}
\]