BEFORE CLASS: Play with toys – how is the energy stored?

SET TIME FOR FIRST HOUR EXAM: 
WED FEB 27 in class (HW1-4)

4th HW DUE IN CLASS WEDNESDAY

class web site
http://www.physics.rutgers.edu/ugrad/272
Charging a capacitor

Let’s compute the work done to charge the capacitor. Transfer the charge between plates one tiny piece dq at a time.

Work done is $\phi(q) \, dq = \left(\frac{q}{C}\right) \, dq$

Total work done is

$$\int_{q'=0}^{q'=q} \phi(q') \, dq' = \int_{q'=0}^{q'=q} \frac{q'}{C} \, dq' = \frac{1}{2} \frac{q^2}{C}$$

Average amount of work done x amount of charge

A capacitor can be charged with a power supply, a battery, or by doing mechanical work (demos)
Capacitor stores energy

The stored energy is in the electric field (energy to charge capacitor is energy to create electric field in region of space)

In parallel plate capacitor, electric field is uniform
\[ E = \sigma/\varepsilon_0 = Q/(A\varepsilon_0) \]
Energy is
\[ U = (1/2) Q^2d/(\varepsilon_0A) = (1/2) E^2d\varepsilon_0A \]
Volume is \( Ad \)
Energy per volume is
\[ U = (1/2) \varepsilon_0 E^2 \]
Connecting capacitors

Capacitors as circuit elements:

two-terminal device
Assume no field outside of “box”

In parallel – top plates at same potential $\phi_1$
Bottom plates at same potential $\phi_2$

\[ Q(\Delta\phi) = Q_1(\Delta\phi) + Q_2(\Delta\phi) = C_1\Delta\phi + C_2\Delta\phi \]
\[ C\Delta\phi = (C_1 + C_2)\Delta\phi \]
\[ C = C_1 + C_2 \]
Connecting capacitors

Capacitors as circuit elements:

two-terminal device
Assume no field outside of “box”

In series – charge $Q_1 = Q_2 = Q$

$\Delta \phi = \frac{Q}{C_1} + \frac{Q}{C_2}$

$\frac{Q}{C} = \frac{Q}{(\frac{1}{C_1} + \frac{1}{C_2})^{-1}}$

$C = (\frac{1}{C_1} + \frac{1}{C_2})^{-1}$
Back to the big picture…

System with conductors – for each conductor, either the potential of the conductor is given OR the charge on the conductor is given

The problem: find the potential function $\phi(\vec{r})$ that satisfies all the rules:

The potential at all points on the surface of the conductor is equal to the given value

$\vec{E}(\vec{r}) = -\nabla \phi(\vec{r})$ is normal to the surface at all points on the surface of the conductor and $\int \epsilon_0 (-\nabla \phi(\vec{r})) \cdot d\vec{A}$ is equal to the given charge
How to solve the problem?
Only ask questions you know the answer to…

START WITH THE SOLUTION AND FORMULATE THE PROBLEMS YOU CAN SOLVE!

Start with two charges: + and –

Look at the potential
Midplane is at potential zero

SO WE CAN FIND THE POTENTIAL OF A SINGLE CHARGE IN THE PRESENCE OF A GROUNDED CONDUCTING PLANE!
For any arrangement of charges, choose the equipotential surface $\phi=0$
Swap a conductor with that surface into the system
Get a system for which we have a solution to the Laplace equation when that conductor is grounded
Play this game for a spherical surface that is an equipotential
For any arrangement of charges, choose the equipotential surface $\phi=0$
Swap a conductor with that surface into the system
Get a system for which we have a solution to the Laplace equation when that conductor is grounded

Put $q$ at $x=s$
At $s'<s$, put $q' = -q\sqrt{s'/s}$
Then $\phi = 0$ on a sphere of radius $\sqrt{ss'}$ centered on the origin

From this, we get the $\phi$ for the system consisting of a grounded conducting sphere of radius $R$ and a point charge $q$ at distance $r$ from the center ($q' = -qR/r$ and $s'=R^2/r$)
System with two conductors: Capacitance matrix

Specify charges $Q_1$ and $Q_2$
$\phi_1(Q_1, Q_2), \phi_2(Q_1, Q_2)$

$Q_1 \neq 0, Q_2 = 0 \rightarrow \phi_1 = P_{11} Q_1, \phi_2 = P_{21} Q_1$
$Q_1 = 0, Q_2 \neq 0 \rightarrow \phi_1 = P_{12} Q_2, \phi_2 = P_{22} Q_2$

“potential coefficients”

Superposition: potentials add
For any choice of $Q_1$ and $Q_2$, get $\phi_1$ and $\phi_2$
$\phi_1 = P_{11} Q_1 + P_{12} Q_2$
$\phi_2 = P_{21} Q_1 + P_{22} Q_2$

“potential coefficient matrix “
System with two conductors

Suppose we choose $\phi_1$ and $\phi_2$ and want to know what $Q_1$ and $Q_2$ should be
Rearrange the equations = invert the matrix

Relate to the capacitance defined before (see Prob 3.23)
Generalize to any number of conductors
Example problem: two concentric spheres
Up to now we have been considering systems where the arrangement of charges is fixed, studying the electric field produced and its description by a potential function. “electrostatics”

Rules include
“Electric field inside a conductor is zero”

Now we allow charges to move:
**New quantities**: current, resistance…
**New rules**: Ohm’s law…