BEFORE CLASS:

Do the CheckUp Question sheet –
use additional paper
try first without looking at your notes
OK not to compute numbers
OK not to evaluate integrals (but nice if you have time)

2nd HOMEWORK ASSIGNMENT IS DUE IN CLASS TODAY

class web site
http://www.physics.rutgers.edu/ugrad/272
Electric Potential $\phi(\vec{r})$

Given electric field $\vec{E}(\vec{r})$ and reference point $r_0$, get potential $\phi(\vec{r})$ by choosing a path from reference point to point of interest and evaluating a line integral (path independent!)

$$\phi(\vec{r}) = -\int \vec{E} \cdot d\vec{l}$$

Given potential $\phi(\vec{r})$, get electric field $\vec{E}(\vec{r})$ by differentiating

$$\vec{E}(\vec{r}) = -\nabla \phi(\vec{r})$$

$\phi(\vec{r})$ contains the same information as $\vec{E}(\vec{r})$
Electric potential for point charge $q$

\[ \phi(\vec{r}) = -\int_{C(\vec{r}_0 \to \vec{r})} \vec{E} \cdot d\vec{l} \]

Choose straight radial path from infinity to $\vec{r}$

\[ \phi(\vec{r}) = -\int_{C(\infty \to \vec{r})} \vec{E} \cdot d\vec{l} = -\int_{\infty}^{r} \frac{kQ}{r^2} dr = \frac{kQ}{r} \]
Electric potential for a uniform field

$$\vec{E}(\vec{r}) = -E_0 \hat{j}$$

produced by an infinite uniform sheet of charge

Choose right angle path from reference point

$$(x_0, y_0)$$

$$\phi(x, y) = -\int_{y_0}^{y} -E_0 \, dy = E_0 (y - y_0)$$
A nonconducting sphere has radius \( R = 2.31 \text{ cm} \) and uniformly distributed charge \( q = +3.50 \text{ fC} \). Take the electric potential at the sphere’s center to be \( V_0 = 0 \). What is \( V \) at radial distance (a) \( r = 1.45 \text{ cm} \) and (b) \( r = R \). (Hint: See Section 23-9.)
Electric potential \( \phi(\vec{r}) = \frac{U(\vec{r})}{q} \)

UNITS: Joules/Coulomb = N m/C = V (volt)

Graphical representation of the electric potential
Contour plot = topographical map
Equipotential lines at equal intervals in \( \phi \)
Important electric field configurations

Electric field of a point charge

Uniform electric field
Equipotential surfaces

On field line diagram, equipotential lines are perpendicular to field lines – no work done

Lines are drawn for values of $\phi(r)$ at equal intervals $0, 20\, \text{V}, 40\, \text{V}, 60\, \text{V} \ldots$

Compare two points on same diagram:
Spacing is smaller where magnitude of field is larger
The three diagrams show equipotentials for three different electric fields covering the same size region of space. In which is the magnitude of the electric field the greatest?
The three diagrams show equipotentials for three different electric fields covering the same size region of space. In which is the magnitude of the electric field the greatest?
The electric potential $\phi(r)$ provides a completely equivalent alternative specification of the electric field

\[
\vec{E}(\vec{r}) \rightarrow \phi(\vec{r})
\]

\[
\phi(\vec{r}) \rightarrow \vec{E}(\vec{r})
\]

One advantage of $\phi$ that we will see shortly: easier to do the integral for the potential, then get the field from $\phi$ than to do the integral for $E$ directly
Relation between E field and potential on a line

\[ \phi(x) = - \int_{x=0}^{x} E(x') \, dx' \quad \text{constant } E: \phi(x) = -E \, x \]

Calculation of field from potential

\[ E(x) = - \frac{d}{dx} \phi(x) \]

Calculation of potential from field (reference point at x=0)

Electric field points “downhill”
8. A graph of the $x$ component of the electric field as a function of $x$ in a region of space is shown in Fig. 24-30. The scale of the vertical axis is set by $E_{xs} = 20.0 \text{ N/C}$. The $y$ and $z$ components of the electric field are zero in this region. If the electric potential at the origin is 10 V, (a) what is the electric potential at $x = 2.0 \text{ m}$, (b) what is the greatest positive value of the electric potential for points on the $x$ axis for which $0 \leq x \leq 6.0 \text{ m}$, and (c) for what value of $x$ is the electric potential zero?

![Graph of $E_x$ vs. $x$](image)
IN THREE DIMENSIONS

\[ \vec{E}(\vec{r}) = -\nabla \phi(\vec{r}) \]

\[ E_x = -\frac{\partial \phi}{\partial x} \]
\[ E_y = -\frac{\partial \phi}{\partial y} \]
\[ E_z = -\frac{\partial \phi}{\partial z} \]

**37 SSM** What is the magnitude of the electric field at the point \((3.00\hat{i} - 2.00\hat{j} + 4.00\hat{k})\) m if the electric potential is given by \(V = 2.00xyz^2\), where \(V\) is in volts and \(x, y,\) and \(z\) are in meters?
Electric field of point charge $Q$

$$\vec{E}(\vec{r}) = kQ / r^2 \hat{r}$$

$$\phi(\vec{r}) = kQ / r$$

$$= kQ / \sqrt{x^2 + y^2 + z^2}$$

$$E_x(\vec{r}) = -\partial \phi / \partial x$$

$$E_y(\vec{r}) = -\partial \phi / \partial y$$

$$E_z(\vec{r}) = -\partial \phi / \partial z$$

$$\vec{E}(\vec{r}) = -\nabla \phi(\vec{r})$$

Check! Electric field of point charge passes test to be a conservative vector field
TRUE, FALSE, or NOT ENOUGH INFO
The vector field shown in the figure could be an electric field produced by some charge distribution.
Charges and Potential

Fixed charge distribution
Get field from charge distribution
Get potential from field

Seems rather tedious…
Charges and Potential

Fixed charge distribution

Potential of a point charge
Sum or integrate over the charges to get $\varphi(\vec{r})$
Charge distribution determines the potential
Potential of a point charge at point $\vec{r}_Q$

$$\phi(\vec{r}) = \frac{kQ}{|\vec{r} - \vec{r}_Q|}$$

Simpler than the expression for electric field

$$\vec{E}(\vec{r}) = \frac{kQ}{|\vec{r} - \vec{r}_Q|^{3/2}} (\vec{r} - \vec{r}_Q)$$
Potential of a collection of point charges

Add up the contributions from each charge

$$\phi(\vec{r}) = \sum_{i} kQ_i / |\vec{r} - \vec{r}_i|$$

CHECKPOINT 4

The figure here shows three arrangements of two protons. Rank the arrangements according to the net electric potential produced at point P by the protons, greatest first.