3rd HOMEWORK ASSIGNMENT IS DUE IN CLASS ON WEDNESDAY

class web site
http://www.physics.rutgers.edu/ugrad/272
Charges and Potential

Fixed charge distribution
Get field from charge distribution
Get potential from field

Seems rather tedious…
Charges and Potential

Fixed charge distribution

Potential of a point charge
Sum or integrate over the charges to get $\varphi(\vec{r})$
Charge distribution determines the potential
Potential of a point charge at point \( \vec{r}_Q \)

\[
\phi(\vec{r}) = \frac{kQ}{|\vec{r} - \vec{r}_Q|}
\]

Simpler than the expression for electric field

\[
\vec{E}(\vec{r}) = \frac{kQ}{|\vec{r} - \vec{r}_Q|^{3/2}} (\vec{r} - \vec{r}_Q)
\]
Potential of a collection of point charges

Add up the contributions from each charge

$$\phi(\vec{r}) = \sum_i kQ_i / |\vec{r} - \vec{r}_i|$$

**CHECKPOINT 4**

The figure here shows three arrangements of two protons. Rank the arrangements according to the net electric potential produced at point $P$ by the protons, greatest first.
Potential of a uniformly charged wire $\lambda$

Slice up into tiny segments and sum $d\phi = (k/r) \, dQ$

Uniform wire on the line

\[ \phi(x) = \int_{s=x_L}^{s=x_R} \frac{k \lambda ds}{|s-x|} \]

For points on the line to the left of the wire

\[ \phi(x) = k \lambda \ln\left(\frac{x-x_R}{x-x_L}\right) \]
Potential of a uniformly charged wire $\lambda$

Slice up into tiny segments and sum $d\phi = (k/r) \, dQ$

Uniform wire in circular arc
$\phi$ at center of the circle $P$

$r=R$ for ALL SEGMENTS

\[
\phi = \int \frac{k dQ}{R} = \frac{k}{R} \int dQ = \frac{kQ}{R}
\]
Potential of a uniformly charged ring $\lambda$

Slice up into tiny segments and sum $d\phi = (k/r)\ dQ$

Uniform ring
$\phi$ at points on the axis

$r = \sqrt{(z^2 + R^2)}$ for ALL SEGMENTS

$$\phi(z) = \frac{k dQ}{\sqrt{z^2 + R^2}} = \frac{k}{\sqrt{z^2 + R^2}} \int dQ = \frac{kQ}{\sqrt{z^2 + R^2}}$$

- Look at the potential far away from the ring
- From here, get to the uniformly charged disk (in book)
Potential of a dipole: dipole moment $p = 2aQ$
Potential “far away” from a dipole

$$\phi(r, \theta) \to \frac{p \cos \theta}{4\pi\varepsilon_0 r^2}$$
Electric field “far away” from a dipole

\[ \phi(r, \theta) \rightarrow \frac{p \cos \theta}{4\pi \varepsilon_0 r^2} \]

\[ E(r, \theta) = \frac{p}{4\pi \varepsilon_0 r^3} (2 \cos \theta \hat{r} + \sin \theta \hat{\theta}) \]
(note that the points in this plot are not “far away”)
Spherically symmetric charge distribution

\[ \rho(\vec{r}) = f(r) \]

Uniformly charged sphere of radius R;
Hollow sphere with concentric cavity…

\[ \vec{E}(\vec{r}) = E(r)\hat{r} \]

Gauss law – integrate the charge inside the relevant sphere
Spherically symmetric charge distribution

\[ \rho(r) = f(r) \]

Uniformly charged sphere of radius R; Hollow sphere with concentric cavity...

Divide charge distribution into spherical shells
Sum or integrate the potentials from the shells

Need to know the potential from a shell
Outside the shell – electric field is the same as if the charge on the shell Q were concentrated at the center.

So the potential is the same as the potential of a point charge Q.

Inside the shell – electric field is zero.
The potential is the same as the potential at the surface of the sphere.
Potential of a spherical shell (charge $Q$, radius $R$)

For $r > R$

$$\vec{E}(\vec{r}) = \frac{kQ}{r^2} \hat{r}$$

$$\phi(\vec{r}) = \int_{r' = r}^{r' = \infty} \frac{kQ}{r'^2} \, dr' = \frac{kQ}{r}$$

Same as for point charge $Q$
(reference point is at $r = \infty$)

For $r < R$

$E$ is zero -> $\phi$ is constant

$$\phi(\vec{r}) = \frac{kQ}{R}$$
Energy to bring a charge $q$ in from infinity

$$q\phi(\vec{r})$$
Energy to assemble a system of charges

Point charges

\[ U = \frac{1}{2} \sum_{j=1}^{N} q_j \sum_{k \neq j} \frac{1}{4\pi \varepsilon_0} \frac{q_k}{r_{jk}} \]
Energy to assemble a system of charges

Point charges
Continuous charge distribution?

\[ U = \frac{1}{2} \sum_{j=1}^{N} q_j \sum_{k \neq j} \frac{1}{4\pi\varepsilon_0} \frac{q_k}{r_{jk}} \rightarrow \frac{1}{2} \int \rho \varphi \, dv \]

Example: energy to assemble a spherical shell

\[ U = \frac{Q^2}{8\pi\varepsilon_0 R} \]

(look in book 1.14 force on a layer of charge)
Charges and Potential

Given the potential \( \varphi(\vec{r}) \)
get the field from the potential
get the charge from the field

Combine to get the charge directly from \( \varphi(\vec{r}) \)

\[ \nabla \cdot E = -\nabla \cdot \nabla \varphi = -\left( \frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} + \frac{\partial^2 \varphi}{\partial z^2} \right) \]

Poisson equation (from Gauss’ law)

\[ \nabla^2 \varphi = -\frac{\rho}{\varepsilon_0} \]
Charges and Potential

Another way to get $\varphi(\vec{r})$ from $\rho(\vec{r})$

Poisson equation

in charge-free regions, Laplace equation

$\nabla^2 \varphi = 0$

$2^{nd}$ order diff equation

“initial conditions” -> “boundary conditions”
Charges and Potential

\[ \varphi(\vec{r}) \] solution of Laplace equation \( \nabla^2 \varphi = 0 \)

A couple of true statements about the potential in a region with zero charge

See p. 87 of Purcell
Systems with conductors
Systems with conductors are DIFFERENT

Charges in conductor move if electric force they feel would move them within the object

Charges rearrange until all charges feel zero electric force for motion within the object

Example: Neutral conductor in uniform external field

GENERAL NOTE: when reading the problem, look for word “conducting” vs “fixed”
Rules for charges and electric fields in electrostatic systems with CONDUCTORS

Fact #1: electric field is ZERO at all points in the material of a conductor

A nonzero field would result in a nonzero force on the free charges at that point and they would move

They are not moving (electrostatics) so there is no force acting on them and the field must be zero.