BEFORE CLASS:

A BIG UNCHARGED WOOD 2x4: electric force?

A BIG CHARGED METAL SPHERE
what happens to nearby objects? To objects that
touch the sphere? Let’s find out!

FIRST HOMEWORK ASSIGNMENT IS DUE IN
CLASS WEDNESDAY

class web site
http://www.physics.rutgers.edu/ugrad/272
Continuous charge distributions (Purcell 1.8)

Line charge density $\lambda$: charge/length (C/m)

Surface charge density $\sigma$: charge/area (C/m$^2$)

Volume density $\rho$: charge/volume (C/m$^3$)

SUMS OVER POINT CHARGES $\rightarrow$ INTEGRALS
TOTAL NET CHARGE \[ \sum_{i} q_i \rightarrow \int_{V} \rho(\vec{r}) \, dV \]

21. A nonconducting spherical shell, with an inner radius of 4.0 cm and an outer radius of 6.0 cm, has charge spread nonuniformly through its volume between its inner and outer surfaces. The volume charge density \( \rho \) is the charge per unit volume, with the unit coulomb per cubic meter. For this shell \( \rho = b/r \), where \( r \) is the distance in meters from the center of the shell and \( b = 3.0 \mu C/m^2 \). What is the net charge in the shell?
31  SSM  ILW  WWW  In Fig. 22-49, a nonconducting rod of length \( L = 8.15 \) cm has a charge \(-q = -4.23 \) fC uniformly distributed along its length. (a) What is the linear charge density of the rod? What are the (b) magnitude and (c) direction (relative to the positive direction of the \( x \) axis) of the electric force produced at point \( P \), at distance \( a = 12.0 \) cm from the rod? What is the electric force magnitude produced at distance \( a = 50 \) m by (d) the rod and (e) a particle of charge \(-q = -4.23 \) fC that replaces the rod?

\[
F(x) = \int_{x'=0}^{x'=L} \frac{kQ\lambda dx}{(x - x')^2}
\]
Our first introduction to looking at the effects of a charge distribution “far away”

\[ r >> R \]
Force exerted by a uniform shell

on a particle outside the shell

A uniform spherical shell of matter attracts a particle that is outside the shell as if all the shell’s mass were concentrated at its center.

A shell of uniform charge attracts or repels a charged particle that is outside the shell as if all the shell’s charge were concentrated at its center.

on a particle inside the shell

A uniform shell of matter exerts no net gravitational force on a particle located inside it.

If a charged particle is located inside a shell of uniform charge, there is no net electrostatic force on the particle from the shell.
Figure 21-17 shows three situations involving a charged particle and a uniformly charged spherical shell. The charges are given, and the radii of the shells are indicated. Rank the situations according to the magnitude of the force on the particle due to the presence of the shell, greatest first.
review potential energy

Consider a thin, spherical, nonconducting shell of radius $R$ with charge $Q$ uniformly distributed on the surface. An electron is then fired directly toward the center of the shell, from point $P$ at distance $r$ from the center of the shell ($r \gg R$). What initial speed $v_0$ is needed for the electron to just reach the shell before reversing direction?
Consider a thin, spherical, nonconducting shell of radius $R$ with charge $Q$ uniformly distributed on the surface. An electron is then fired directly toward the center of the shell, from point $P$ at distance $r$ from the center of the shell ($r \gg R$). What initial speed $v_0$ is needed for the electron to just reach the shell before reversing direction?

The force on the electron everywhere OUTSIDE the shell is the same as from a point charge $Q$ at the origin.
# Gravity vs electric force

<table>
<thead>
<tr>
<th>Gravity</th>
<th>electric force</th>
</tr>
</thead>
<tbody>
<tr>
<td>Acts at a distance</td>
<td>Acts at a distance</td>
</tr>
<tr>
<td>Attractive</td>
<td>Attractive AND repulsive</td>
</tr>
<tr>
<td>Proportional to mass</td>
<td>proportional to charge</td>
</tr>
<tr>
<td>Central</td>
<td>central</td>
</tr>
<tr>
<td>Inverse square law</td>
<td>inverse square law</td>
</tr>
<tr>
<td>Mass is not conserved</td>
<td>charge is conserved</td>
</tr>
<tr>
<td>Mass is not quantized</td>
<td>charge is quantized in units of e</td>
</tr>
</tbody>
</table>

\[ e = 1.602 \times 10^{-19} \text{ C.} \]

Purcell 1.2-1.3
Can there be an electric force between a charged object and a neutral object?
All matter is made of charged particles

Nuclei (protons) are positively charged
Electrons are negatively charged

Net charge on an object can be zero (neutral)
positive or negative (charged)

insulators vs conductors
Can there be an electric force between a charged object and a neutral object?

Object can have charged regions and still have net charge = 0
Simple example where net force is not zero

\[ +q \quad -q \quad +q \]
Materials are made out of charged particles:
Positively charged atomic nuclei
Negatively charged electrons

Nuclei move away from external positive charge
electrons move towards from external charge
External charge induces rearrangement of charges in an object.

[Diagram: A line with positive and negative charges labeled, with arrows indicating movement of electrons and atomic nuclei.]
Can there be an electric force between a charged object and a neutral object?

Induced charge rearrangement leads to a net force
What is the direction of the force for a negative external charge?

- $-q$

(A) towards the external charge  
(B) away from the external charge  
(C) the force is zero  
(D) cannot determine without more information
What is the direction of the force for a negative external charge?

- \(-q\)

(A) towards the external charge  
(B) away from the external charge  
(C) the force is zero  
(D) cannot determine without more information

ANSWER WITH THE DEMO!  
(need the other kind of stick with opposite charge)
What is the direction of the force for a negative external charge?

(A) towards the external charge
(B) away from the external charge
(C) the force is zero
(D) cannot determine without more information
Coulomb’s law for a point charge \( (k = 1/(4\pi\varepsilon_0)) \) + superposition

Find force on charge \( q \) due to any fixed arrangement of charges

Collection of point charges
Continuous charge distributions
$q_1$
\[ \vec{F} = q \left( \frac{kq_1}{r^2} \right) \hat{r} \]

\[ \vec{r} = \text{vector from } q_1 \text{ to } q \]

\[ \hat{r} = \vec{r} / |\vec{r}| \]
For each point $\vec{r}$ there is a vector “vector field”

Electric field $\vec{E}(\vec{r})$

Force on particle $q$ at $\vec{r}$: $\vec{F} = q\vec{E}(\vec{r})$

Here $\vec{E}(\vec{r}) = \frac{kq_1}{r^2} \hat{r}$
Graphical representation of vector field by field lines
Electric field lines point in the direction of E at any point
Spacing of lines decreases as magnitude of E increases
Electric field is force /charge, units of N/C

Charge q modifies the space around it
Not just a mathematical construct, but a physical reality
For each point $\vec{r}$ there is a vector “vector field”
Electric field $\vec{E}(\vec{r})$
Force on particle $q$ at $\vec{r}$: $\vec{F} = q\vec{E}(\vec{r})$
Inverse square law
+ Superposition for E

Find electric field due to any fixed arrangement of charges

Collection of point charges
Continuous charge distributions

From this, find the force on a charge \( q \) at \( r \)

\[
\vec{F} = q\vec{E}(\vec{r})
\]
Electric field of a uniformly charged spherical shell (get this from the shell theorem)
Electric field of a Van de Graff generator
A single charge on a line

\[ Q, x_0 \]
A single charge on a line

\[ E = \frac{-kQ}{(x-x_0)^2} \quad Q, \quad x_0 \quad E = \frac{kQ}{(x-x_0)^2} \]

\( E>0 \) field points to the right
\( E<0 \) field points to the left
Two charges on a line

$Q_1, x_1$  $Q_2, x_2$
Two charges on a line

\[ E(x) = -\frac{kQ_1}{(x-x_1)^2} - \frac{kQ_2}{(x-x_2)^2} \text{ left} \]
\[ E(x) = +\frac{kQ_1}{(x-x_1)^2} - \frac{kQ_2}{(x-x_2)^2} \text{ middle} \]
\[ E(x) = +\frac{kQ_1}{(x-x_1)^2} + \frac{kQ_2}{(x-x_2)^2} \text{ right} \]
Two charges on a line

If $Q_2 = 4Q_1$ and $x_2 - x_1 = d$, where is the E field equal to 0?
The figure shows four systems in which charges are arranged on a line. In which system is the magnitude of the electric field at the central point GREATEST?
The figure shows four systems in which charges are arranged on a line. In which system is the magnitude of the electric field at the central point GREATEST?
Definition of dipole

Electric field of the dipole?
Electric field at points on the x-axis

Q

-\( Q \)
Electric field at points on the x-axis
Electric field at points on the y-axis
Electric field of continuous charge distributions

**Line charge density** $\lambda$: charge/length (C/m)
Surface charge density $\sigma$: charge/area (C/m$^2$)
Volume density $\rho$: charge/volume (C/m$^3$)

**Step by step**
Parametrize the curve
Do the geometry to get distance and angles at each point on curve
Set up integral
Do the integral
E field of an infinite uniform line charge
Rod of charge $Q$ is bent into semicircle of radius $R$.
Linear charge density $\lambda = Q/\pi R$
Find electric force on charge $q$ at any point on line through the center

Parametrize the curve by angle $\theta$
$dQ = \lambda R \, d\theta$

Vertical component vanishes by symmetry

Horizontal component at center
Integrate $(\sin \theta) \, dF$ from $\theta = 0$ to $\theta = \pi$

$E = 2 \, k \lambda / R$ to the right
Electric field of uniformly charged ring on its central axis

\[ F = \frac{k q Q z}{(z^2 + R^2)^{3/2}} \]

For \( z \gg R \), looks like point charge \( Q \)

The perpendicular components just cancel but the parallel components add.
Force due to a surface charge distribution

Example: uniform spherical shell
Force due to a volume charge distribution

Example: uniform hemisphere (in book)
Charge distribution determines electric field
sum, line integral, surface integral, volume integral

--shouldn’t there be an easier way to prove the shell theorem?
--does the electric field determine the charge distribution?
--how do you find the charge distribution from the electric field?
Charge distribution determines electric field sum, line integral, surface integral, volume integral

--shouldn’t there be an easier way to prove the shell theorem?
--does the electric field determine the charge distribution?
--how do you find the charge distribution from the electric field?

REFORMULATION OF COULOMB’S LAW

\[ \oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enc}}}{\varepsilon_0} \]

GAUSS’ LAW

\[ \vec{\nabla} \cdot \vec{E} = \frac{\rho(\vec{r})}{\varepsilon_0} \]
What is a surface?

A surface is a set of points in three-dimensional space satisfying one equation (and optionally some inequalities)

\{(x,y,z) \text{ such that } z = 1\} \text{ (plane)}
\{(x,y,z) \text{ such that } x^2+y^2+z^2 = 100\} \text{ (sphere)}
\{(x,y,z) \text{ such that } z = 1 \text{ and } x^2+y^2 \leq 100\} \text{ (disk)}

Open surface: has a boundary

Closed surface: has no boundary, has an inside and an outside