Wire coil with N turns

Current \( i \) through coil produces a magnetic flux \( \Phi_B \) through ITSELF: \( \Phi_B = N \oint \vec{B} \cdot d\vec{A} = Li \)

Changing current \( i(t) \) through coil produces a changing magnetic flux \( \Phi_B(t) \) through ITSELF and induced EMF: \( \Phi_B(t) = Li(t) \) and \( \mathcal{E} = -d\Phi_B/dt = -L \, di/dt \)

\[ \rightarrow i \]

L is called the self-inductance
Units: 1 T m\(^2\)/A = 1 H (1 henry)

To find the sense of the induced emf, use Lenz’s law
Loop with resistance $R=0$ and current $i$

If the current changes, the magnetic flux will change
$\Phi(t) = L \, i(t)$

Effect is to create electric fields in the loop
These fields induce current that opposes the change in current
Like inserting a battery with $\mathcal{E} = |d\Phi(t)/dt| = = L di(t)/dt$
A new circuit element: the inductor

You can still apply the loop rule If as you go through the inductor in the direction of the current arrow, you count a voltage difference \( V = -L \frac{di(t)}{dt} \)

Note that if \( i \) does not vary with time, there is NO voltage difference
Analyzing time-dependent circuits
• currents that change with time

• Resistors whose currents change with time
• capacitors whose charges change with time
• Inductors whose currents change with time
• Emf sources whose emf changes with time

TODAY: single loop circuits
Charging RC circuit (includes $E$)
Initial condition $Q(t=0) = 0$

Discharging RC circuit
Initial condition $Q(t=0) = Q_0$

Charging RL circuit (includes $E$)
Initial condition $i(t) = 0$

Discharging RL circuit
Initial condition $i(t=0) = i_0$

LC circuit
Initial condition $Q(t=0) = Q_0$

RLC circuit
Initial condition $Q(t=0) = Q_0$
Analyzing single-loop time-dependent circuits

Choose the direction of the current and label the capacitor charge

Snapshot at any time t
• Apply the loop rule (emf source, resistor, capacitor, inductor)
  Go through emf source from small to large plate: +E
  Go through R in direction of i: -iR
  Go through C from –Q to +Q: +Q/C
  Go through L in direction of current: -Ldi/dt

• Current i(t) through capacitor = dQ(t)/dt
Analyzing single-loop time-dependent circuits

Choose the direction of the current and label the capacitor charge

Snapshot at any time \( t \)
- Apply the loop rule (emf source, resistor, capacitor, inductor)
- Current \( i(t) \) through capacitor = \( \frac{dQ(t)}{dt} \)

For RC and RL circuits
get a (simple) first-order differential equation
“Initial condition” = current (and charge) at \( t=0 \)
Solve to get currents (and charge) at all later times \( t \)

Make a graph of \( i(t) \)
Discharging RC circuit

Initial condition \( Q(t=0) = Q_0 \)

Loop rule: \( \frac{Q(t)}{C} + i(t)R = 0 \)
\( \frac{Q(t)}{C} + R \frac{dQ(t)}{dt} = 0 \)

Solution: \( Q(t) = Q_0 e^{-t/(RC)} \)
In graph, \( RC = 2 \) ms

\( i(t) = -\left(\frac{Q_0}{RC}\right) e^{-t/(RC)} \)

As the capacitor discharges, the voltage drop across the resistor decreases and the current decreases
At long times, the charge and the current are zero

Energy stored in capacitor at \( t=0 \) is dissipated by the resistor
STEP BY STEP FOR SINGLE LOOP CIRCUITS

1. Choose the current direction and label the capacitor charges
2. Apply loop rule to write the differential equation
3. Solve the differential equation for the given initial condition
4. Look at the graph of $i(t)$
5. Talk about short time and long time behavior, time scale
6. Talk about energy
Charging RL Circuit

Switch closed to “a” position at t=0

\[\begin{align*}
-iR - L \frac{di}{dt} + \mathcal{E} &= 0 \\
\end{align*}\]

\[i = \frac{\mathcal{E}}{R} \left(1 - e^{-Rt/L}\right),\]

\[\tau_L = \frac{L}{R} \quad \text{(time constant)}\]

Fig. 30-15  An RL circuit. When switch S is closed on a, the current rises and approaches a limiting value $\mathcal{E}/R$.

Initially, an inductor acts to oppose changes in the current through it. A long time later, it acts like ordinary connecting wire.
Energy Stored in an Inductor: consider a charging RL circuit

\[ \mathcal{E} = L \frac{di}{dt} + iR, \]
\[ \frac{dU_B}{dt} = Li \frac{di}{dt}. \]
\[ \mathcal{E}i = Li \frac{di}{dt} + i^2R, \]

\[
\int_0^i U_B \, dU_B = \int_0^i Li \, di
\]

\[ U_B = \frac{1}{2} Li^2 \quad \text{(magnetic energy)}, \]

To increase the current from zero, the battery has to do work against the emf induced by the change in the current.

**Fig. 30-16** The circuit of Fig. 30-15 with the switch closed on a. We apply the loop rule for the circuit clockwise, starting at x.
Charging RC circuit (includes \( E \))
Initial condition \( Q(t=0) = 0 \)

Discharging RC circuit
Initial condition \( Q(t=0) = Q_0 \)

Charging RL circuit (includes \( E \))
Initial condition \( i(t) = 0 \)

Discharging RL circuit
Initial condition \( i(t=0) = i_0 \)

LC circuit
Initial condition \( Q(t=0) = Q_0 \)

RLC circuit
Initial condition \( Q(t=0) = Q_0 \)
LC circuit
Initial condition $Q(t=0) = q_0$

Loop rule: $Q(t)/C + L\frac{di(t)}{dt} = 0$
$Q(t)/C + L \frac{d^2Q(t)}{dt^2} = 0$
$2^{nd}$ order differential equation

Solution: $Q(t) = q_0 \cos(t/(LC)^{1/2})$

$i(t) = -(q_0/(LC)^{1/2}) \sin(t/(LC)^{1/2})$

Oscillation with angular frequency $\omega = 1/(LC)^{1/2}$

As the capacitor discharges, the current builds up
As the capacitor charges, the current drops
Energy $Q^2/(2C) + Li^2/2$ is conserved
Alternates between all in the capacitor and all in the inductor
RLC circuit

Initial condition $Q(t=0) = q_0$

Loop rule: $Q(t)/C + Ri(t) + Ld(i(t)/dt = 0$

$Q(t)/C + RdQ(t)/dt + L d^2Q(t)/dt^2 = 0$

2nd order differential equation

Damped oscillator – discuss energy

Solution discussed in Purcell 8.1:

Increase $R$ from $R=0$

- Underdamped
- Critically damped
- Overdamped

Underdamped when $\omega > (R/2L)$

$Q(t) = Q_0 e^{-Rt/2L} \cos(\omega' t+\phi)$ where $\omega' = (\omega^2 - (R/2L)^2)^{1/2}$